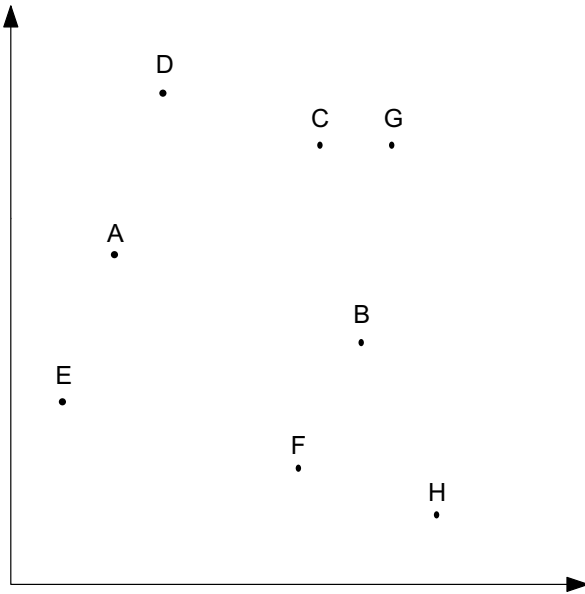**Midterm 1 Practice Problems**   *(find and work with a partner)*
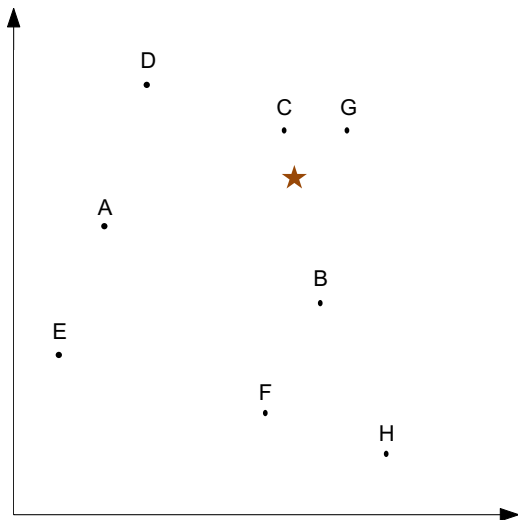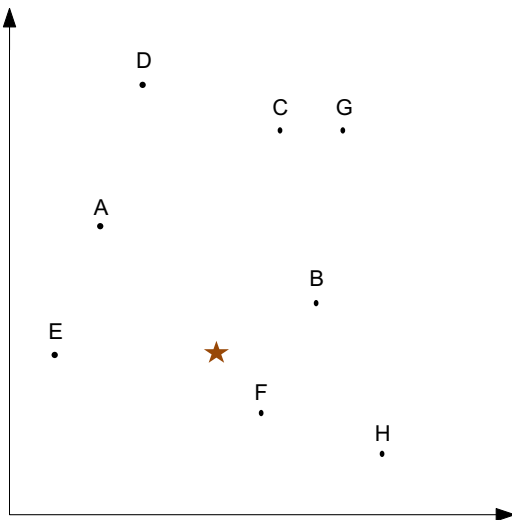
1. *KD Trees.*

   (a) Use the KD tree algorithm to create a KD tree for the example training data below, starting
       with dimension 0 (x-axis here).

   (b) Does the KD tree algorithm create balanced or imbalanced trees?

   (c) For the query point below, show which training points are visited and what is the nearest
       neighbor.

   (d) How can the idea of nearest neighbors be used in regression problems? Classification prob-
       lems? Multi-class classification? Missing data?

2. *Gradient Descent.* Say we have the function $f(w) = w^2 - 6w + 9$ and we want to find the value of w that *minimizes* this function.

   (a) If we used gradient descent with $\alpha = 1$ and an initial value of $w = 0$, what are the first three gradient descent updates?

   (b) If we changed $\alpha = 0.1$, what are the first two gradient descent updates?

3. *Logistic Regression.* Say we have $p = 1$ and two training examples: $(x_1, y_1) = (3, 0)$ and $(x_2, y_2) = (7, 1)$, and we would like to fit a logistic model to this dataset.

   (a) Draw these two examples on a coordinate system and sketch the logistic function that would fit them. What is the linear decision boundary that should be created?

   (b) In terms of $h_{\boldsymbol{b}}(x)$, write out the likelihood $L(\boldsymbol{b})$ of observing this data (don't need to simplify).

   (c) Say in our SGD method, we choose to analyze (x_2, y_2) first. Before starting SGD, we set $w_0 = 0$ and $w_1 = 0$. After analyzing (x_2, y_2), what are w_0 and w_1? Choose $\alpha = 0.1$.

   (d) Next we consider (x_1, y_1). What are w_0 and w_1 be after this second data point? At this point we have finished *one* iteration of SGD.