Logistic Regression and Regularization

1. A key step in our derivation of the SGD updates for logistic regression was the fact that $g^{\prime}(z)=$ $g(z)(1-g(z))$, where $g(z)=\frac{1}{1+e^{-z}}$. This allowed us to cancel out the terms in the denominators. Compute the derivative of $g(z)$ to demonstrate this fact. What does $g^{\prime}(z)$ tend to as $z \rightarrow \infty$ ? As $z \rightarrow-\infty$ ?
2. The confusion matrices below show hiring predictions separated by demographic group (non-men and men). To put this in the context of our fairness regularization setup, identify $Y \in\{0,1\}$ and $A \in\{0,1\}$.

| Non-men |  |  |  | Men |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Predicted <br> don't hire | Predicted <br> Do hire |  |  | Predicted <br> don't hire | Predicted <br> DO hire |
| Test Label <br> wasn't <br> hired | 542 | 170 |  | Test Label <br> wasn't <br> hired | 1598 | 430 |
| Test Label <br> WAS hired | 23 | 56 |  | Test Label <br> WAS hired | 340 | 190 |

3. Compute the demographic parity:

$$
\frac{P(\hat{Y}=1 \mid A=1)}{P(\hat{Y}=1 \mid A=0)}
$$

4. Compute the equalized odds:

$$
\frac{P(\hat{Y}=1 \mid A=1, Y=y)}{P(\hat{Y}=1 \mid A=0, Y=y)} \quad \text { for } \quad y \in\{0,1\}
$$

