

# **CS 106**

# **INTRODUCTION TO**

# **DATA STRUCTURES**

**SPRING 2020**

**PROF. SARA MATHIESON**

**HAVERFORD COLLEGE**

# ADMIN

- **Midterm 1 in-class on Thursday**
  - Create one-page (front & back) “cheat-sheet”
- **Office Hours TODAY! 4:30-6pm (H110)**
- **Remind me to hand back Handout 12 and Lab 2**
- **We DO have lab this week and attendance is still required. You do NOT need to do anything in advance though, you may begin Lab 4 during lab.**
  - Lab 4 will be posted Wed or Thurs

# **MAR 3 OUTLINE**

- **Queues (theory and implementation)**
- **Review arrays and ArrayLists**
- **Review Nodes and Linked Lists**
- **Practice Problems**

# MAR 3 OUTLINE

- **Queues (theory and implementation)**
- Review arrays and ArrayLists
- Review Nodes and Linked Lists
- Practice Problems

# QUEUES

How would you want a data structure to work for waiting in line at a store?

What is the rate of **input** is different than the rate of **output**?

Example: people show up to the DMV at random times, but processing takes about the same time for each person

Define an abstract data type (ADT).

# THE QUEUE ADT

**Insertions and deletions are First In First Out (FIFO)**

- Insert at the back
- Delete from the front

## **Operations:**

- `enqueue(Object)`
- `Object dequeue()`
- `Object first()`
- `int size()`
- `boolean isEmpty()`

# IMPLEMENTING A QUEUE

**Brainstorm: using the data structures we know about, how could we implement this ADT?**

# IMPLEMENTING A QUEUE

**Brainstorm: using the data structures we know about, how could we implement this ADT?**

Many ways to implement a Queue! Underneath, we can use:

- \* Arrays
- \* Lists
- \* Stacks



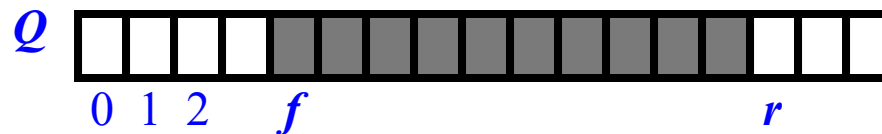
# ARRAY-BASED QUEUE IMPLEMENTATION

An array of size  $n$  in a circular fashion

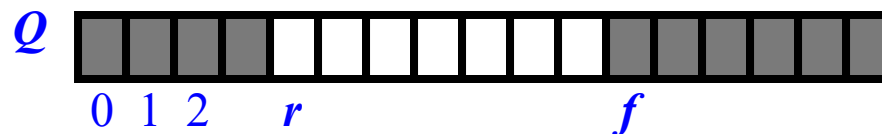
Two `ints` to track front and size

- `f`: index of the front element
- `size`: number of stored elements

normal configuration



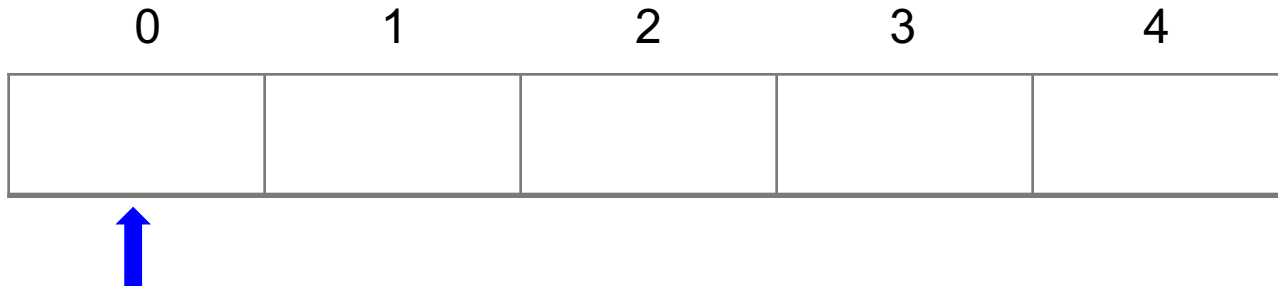
wrap-around configuration (circular)



# EXAMPLE

→ `Queue<Integer> testQ = new ArrayQueue<Integer>(5);`  
`testQ.enqueue(3);`  
`testQ.enqueue(4);`  
`testQ.enqueue(5);`  
`testQ.enqueue(2);`  
`testQ.enqueue(1);`  
`testQ.dequeue();`  
`testQ.dequeue();`  
`testQ.enqueue(-9);`  
`testQ.dequeue();`  
`testQ.dequeue();`  
`testQ.dequeue();`  
`testQ.dequeue();`  
`testQ.enqueue(-8);`

Size: 0

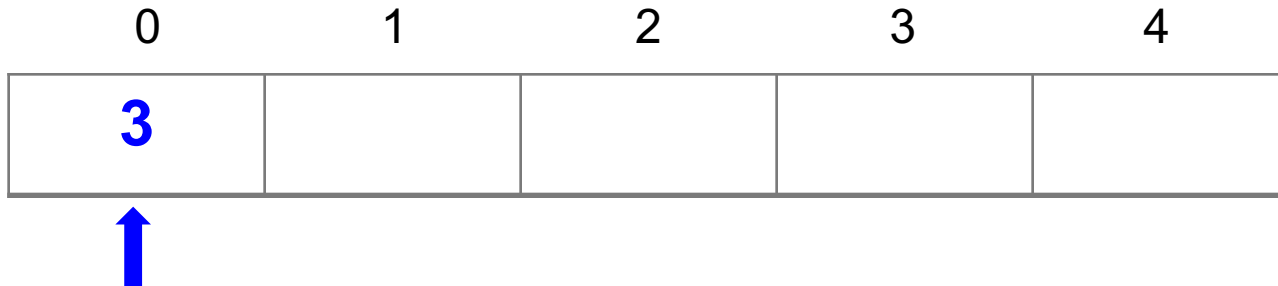


*Arrow is front*

# EXAMPLE

```
Queue<Integer> testQ = new ArrayQueue<Integer>(5);  
testQ.enqueue(3);  
testQ.enqueue(4);  
testQ.enqueue(5);  
testQ.enqueue(2);  
testQ.enqueue(1);  
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testQ.dequeue();  
testQ.enqueue(-9);  
testQ.dequeue();  
testQ.dequeue();  
testQ.dequeue();  
testQ.dequeue();  
testQ.enqueue(-8);
```

Size: 1

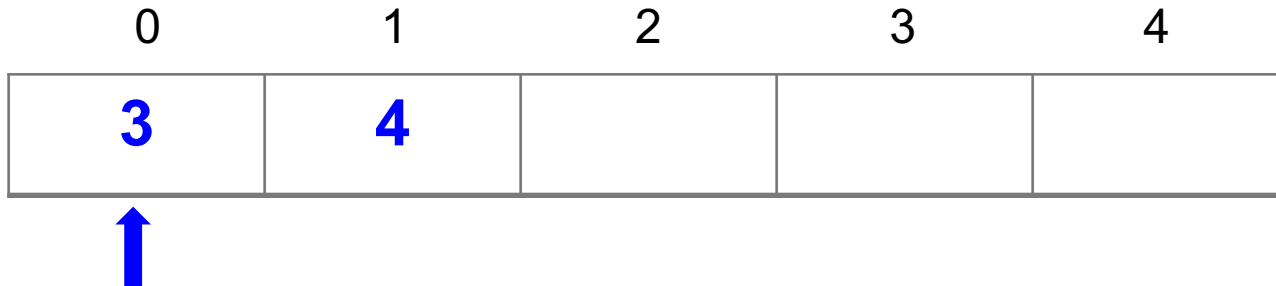


*Arrow is front*

# EXAMPLE

```
Queue<Integer> testQ = new ArrayQueue<Integer>(5);  
testQ.enqueue(3);  
→ testQ.enqueue(4);  
testQ.enqueue(5);  
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testQ.dequeue();  
testQ.enqueue(-9);  
testQ.dequeue();  
testQ.dequeue();  
testQ.dequeue();  
testQ.dequeue();  
testQ.enqueue(-8);
```

Size: 2

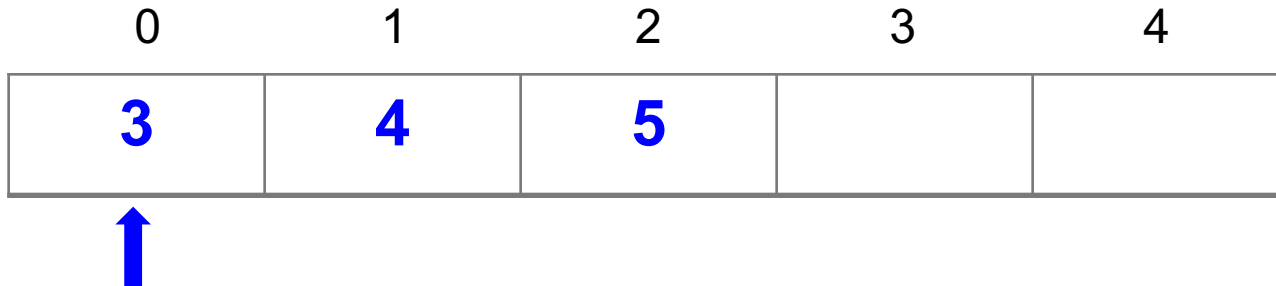


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# EXAMPLE

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testQ.dequeue();  
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testQ.enqueue(-8);
```

Size: 3

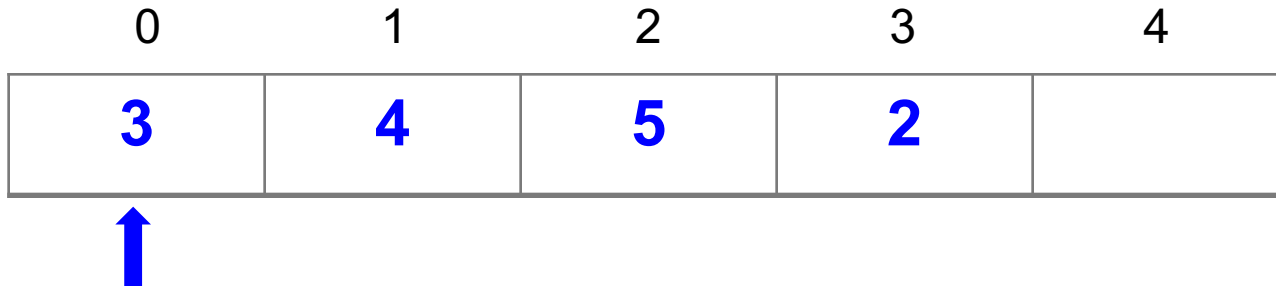


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# EXAMPLE

```
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testQ.dequeue();  
testQ.dequeue();  
testQ.enqueue(-9);  
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testQ.dequeue();  
testQ.dequeue();  
testQ.dequeue();  
testQ.enqueue(-8);
```

Size: 4

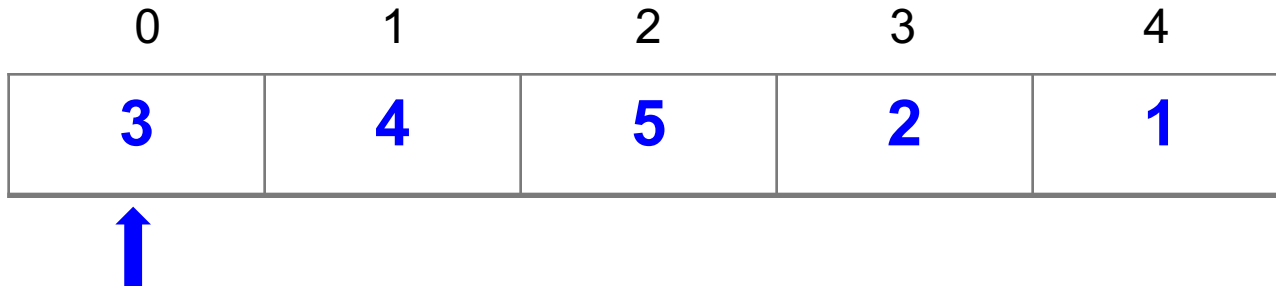


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# EXAMPLE

```
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testQ.enqueue(-9);  
testQ.dequeue();  
testQ.dequeue();  
testQ.dequeue();  
testQ.dequeue();  
testQ.enqueue(-8);
```

Size: 5

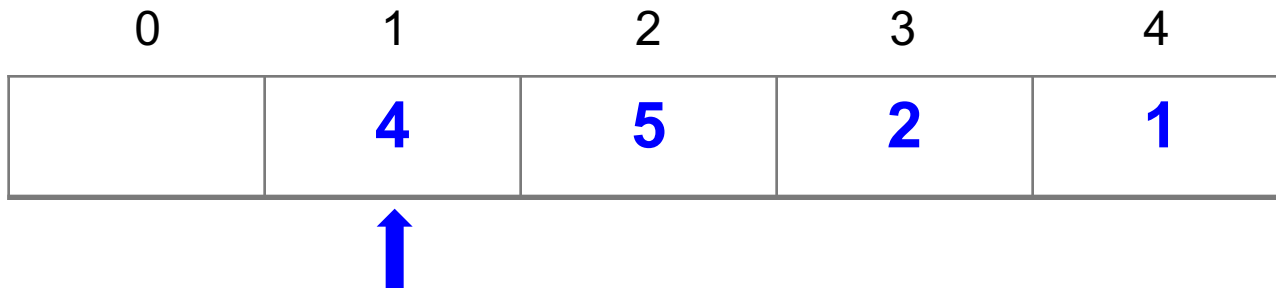


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# EXAMPLE

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```

Size: 4



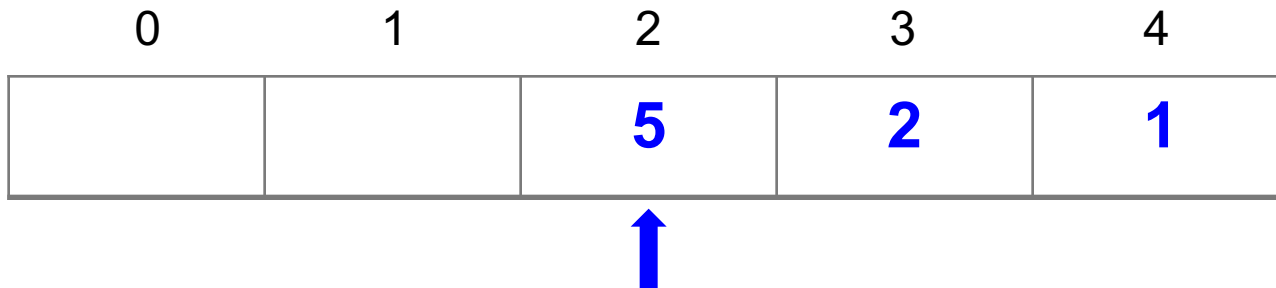
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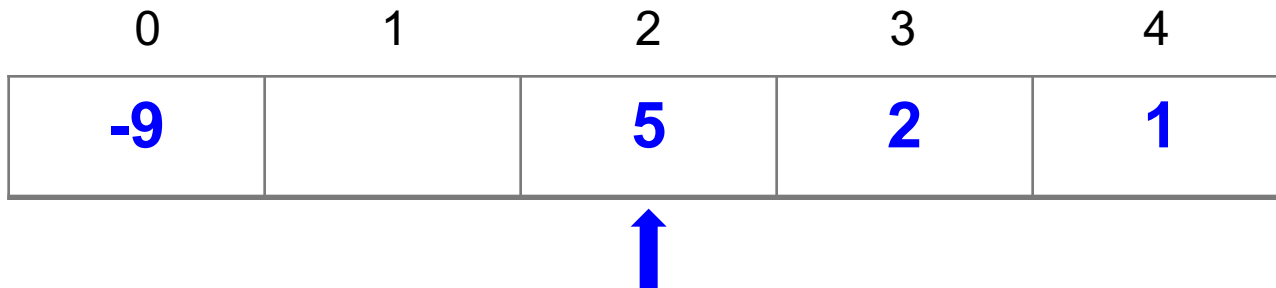


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```

Size: 4

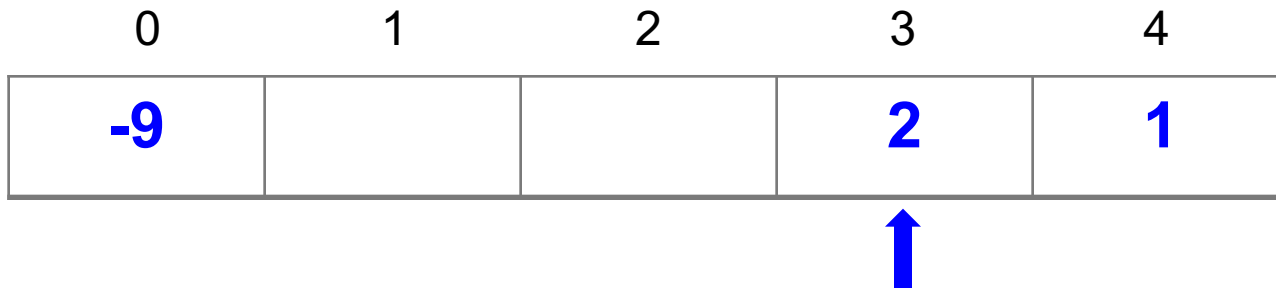


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testQ.dequeue();  
testQ.enqueue(-8);
```

Size: 3



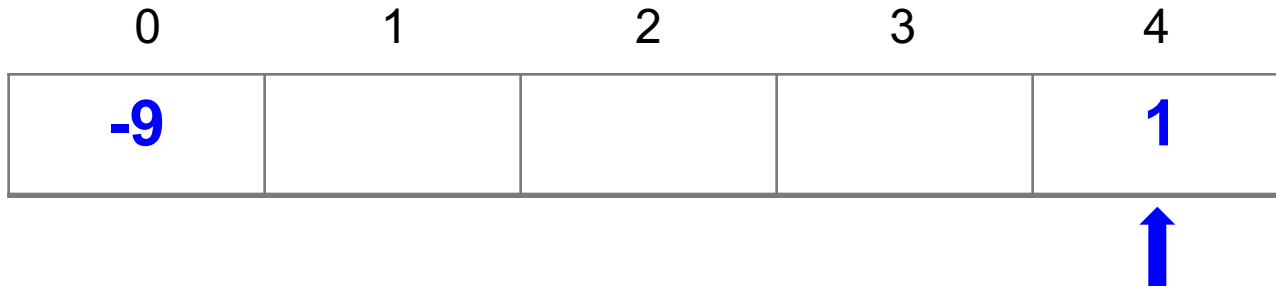
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testQ.enqueue(-8);
```



Size: 2

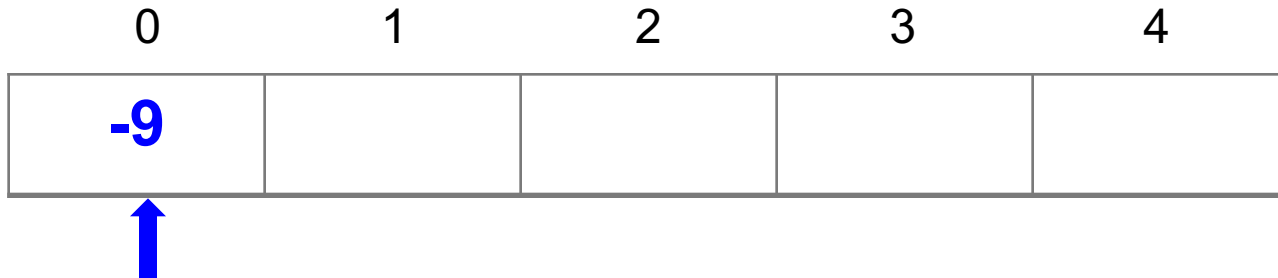


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# EXAMPLE

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testQ.dequeue();  
testQ.enqueue(-9);  
testQ.dequeue();  
testQ.dequeue();  
→ testQ.dequeue();  
testQ.dequeue();  
testQ.enqueue(-8);
```

Size: 1

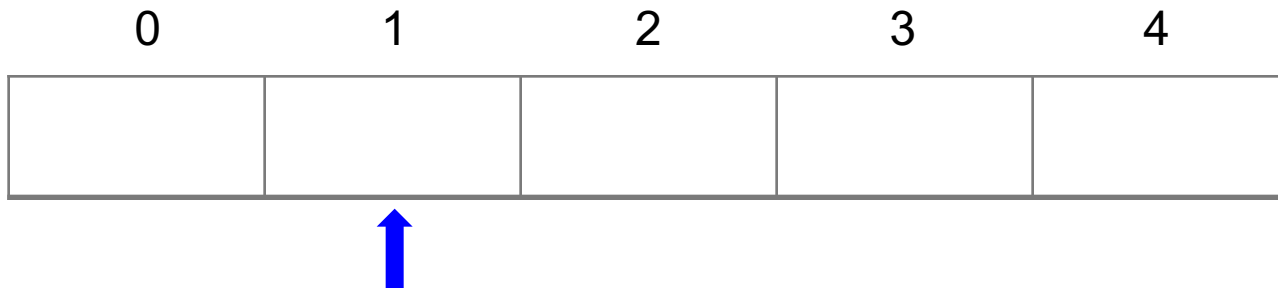


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testQ.dequeue();  
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testQ.dequeue();  
testQ.dequeue();  
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```

Size: 0

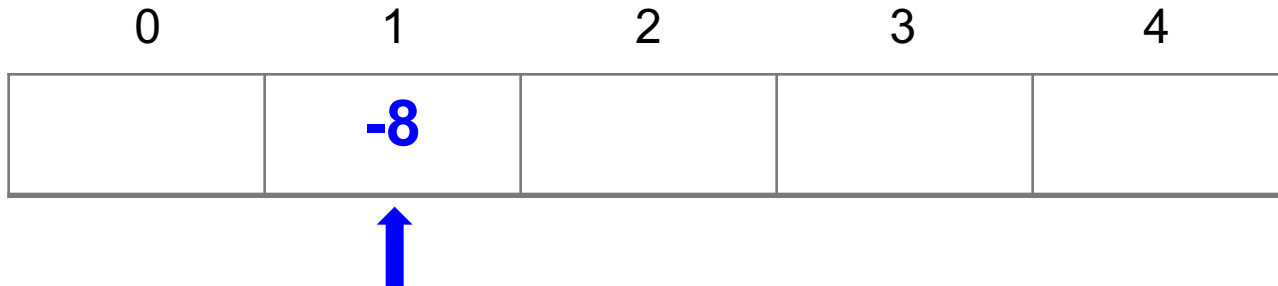


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testQ.dequeue();  
testQ.dequeue();  
testQ.dequeue();  
testQ.enqueue(-8);
```

Size: 1



*Arrow is front*

# QUEUE WITH CIRCULAR ARRAY

```
public class ArrayQueue<E> implements Queue<E> {

    public static final int CAPACITY = 1000;

    private int f;
    private int size;
    private E[] data;

    public ArrayQueue() {

    }

    @SuppressWarnings("unchecked")
    public ArrayQueue(int capacity) {

    }

    public int size() {

    }

    public boolean isEmpty() {

    }

    public E first() throws EmptyQueueException {

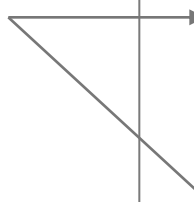
    }

}
```



# QUEUE WITH CIRCULAR ARRAY

*Two constructors (allow  
user to select capacity, or  
use default.*



```
public class ArrayQueue<E> implements Queue<E> {  
  
    public static final int CAPACITY = 1000;  
  
    private int f;  
    private int size;  
    private E[] data;  
  
    public ArrayQueue() {  
        this(CAPACITY);  
    }  
  
    @SuppressWarnings("unchecked")  
    public ArrayQueue(int capacity) {  
        f = 0;  
        size = 0;  
        data = (E[]) new Object[capacity];  
    }  
  
    public int size() {  
  
    }  
  
    public boolean isEmpty() {  
  
    }  
  
    public E first() throws EmptyQueueException {  
  
    }  
  
}
```

# QUEUE WITH CIRCULAR ARRAY

```
public class ArrayQueue<E> implements Queue<E> {

    public static final int CAPACITY = 1000;

    private int f;
    private int size;
    private E[] data;

    public ArrayQueue() {
        this(CAPACITY);
    }

    @SuppressWarnings("unchecked")
    public ArrayQueue(int capacity) {
        f = 0;
        size = 0;
        data = (E[]) new Object[capacity];
    }

    public int size() {
        return size;
    }

    public boolean isEmpty() {

    }

    public E first() throws EmptyQueueException {

    }

}
```

# QUEUE WITH CIRCULAR ARRAY

```
public class ArrayQueue<E> implements Queue<E> {

    public static final int CAPACITY = 1000;

    private int f;
    private int size;
    private E[] data;

    public ArrayQueue() {
        this(CAPACITY);
    }

    @SuppressWarnings("unchecked")
    public ArrayQueue(int capacity) {
        f = 0;
        size = 0;
        data = (E[]) new Object[capacity];
    }

    public int size() {
        return size;
    }

    public boolean isEmpty() {
        return size == 0;
    }

    public E first() throws EmptyQueueException {

    }

}
```

# QUEUE WITH CIRCULAR ARRAY

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    @SuppressWarnings("unchecked")
    public ArrayQueue(int capacity) {
        f = 0;
        size = 0;
        data = (E[]) new Object[capacity];
    }

    public int size() {
        return size;
    }

    public boolean isEmpty() {
        return size == 0;
    }

    public E first() throws EmptyQueueException {
        if (isEmpty()) {
            throw new EmptyStackException();
        }
        return data[f];
    }
}
```

# QUEUE WITH

```
public void enqueue(E element) throws FullQueueException {
```

}

```
public E dequeue() throws EmptyQueueException {
```

}

# QUEUE WITH CIRCULAR ARRAY

```
public void enqueue(E element) throws FullQueueException {
    if (size == data.length) {
        throw new FullQueueException();
    }

    public E dequeue() throws EmptyQueueException {
        if (isEmpty()) {
            throw new EmptyStackException();
        }

    }
```

# QUEUE WITH CIRCULAR ARRAY

```
public void enqueue(E element) throws FullQueueException {
    if (size == data.length) {
        throw new FullQueueException();
    }

    int end = (f + size) % data.length;
    data[end] = element;

    size += 1;
}

public E dequeue() throws EmptyQueueException {
    if (isEmpty()) {
        throw new EmptyStackException();
    }
}

}
```

# QUEUE WITH CIRCULAR ARRAY

```
public void enqueue(E element) throws FullQueueException {
    if (size == data.length) {
        throw new FullQueueException();
    }

    int end = (f + size) % data.length;
    data[end] = element;

    size += 1;
}

public E dequeue() throws EmptyQueueException {
    if (isEmpty()) {
        throw new EmptyStackException();
    }
    E result = data[f];
    data[f] = null; // optional
    f = (f + 1) % data.length;

    size -= 1;
    return result;
}
```

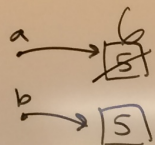


# DESIGNING DATA STRUCTURES

1. Make a **Course** object that can store a name and list of students. Include relevant constructors, getters, and setters.
2. Make a **LimitedEnrollmentCourse** that has a cap on the number of students who can enroll. Have it inherit from **Course**.
3. Make **addStudent**, **removeStudent**, and **getEnrolled** methods that correctly handle limited versus unlimited enrollment.

*Extra practice!*

int a = 5  
int b = a  
a = 6



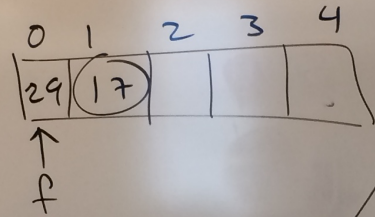
## Primitive types (immutable)

- int
- double
- char
- boolean

Strings are  
immutable!

but not primitive!

~~ArrayList~~ <int>  
                  <Integer>



dequeue()  
dequeue()

Static  
(only one  
for whole  
class)

size = 3 ← class name.  
Integer. parseInt(...)  
enqueue(17)

$$(f + \text{size}) \% \text{length}$$
$$(3 + 3) \% 5$$
$$= 1$$

# MAR 3 OUTLINE

- Queues (theory and implementation)
- **Review arrays and ArrayLists**
- Review Nodes and Linked Lists
- Practice Problems

# ARRAYS

- **Fixed length**
  - Pro: all operations  $O(1)$
  - Con: cannot resize or move around elements easily
- **Declare**

```
String[] words;
```
- **Initialize (allocate)**

```
words = new String[3];
```

# ARRAYS

- **Fixed length**
  - Pro: all operations  $O(1)$
  - Con: cannot resize or move around elements easily

- **Declare**

```
String[] words;
```

- **Initialize (allocate)**

```
words = new String[3];
```

- **“set”:**

```
words[1] = "spring";  
words[2] = "break";
```

- **“get”:**

```
String w = words[2];
```

- **length**

```
int len = words.length;
```

# ARRAYS

```
String[] otherWords = words;  
otherWords[0] = "welcome";
```

- **Fixed length**

*Q: what is happening here??*

- Pro: all operations  $O(1)$
- Con: cannot resize or move around elements easily

- **Declare**

```
String[] words;
```

- **Initialize (allocate)**

```
words = new String[3];
```

- **“set”:**

```
words[1] = "spring";  
words[2] = "break";
```

- **“get”:**

```
String w = words[2];
```

- **length**

```
int len = words.length;
```

# ARRAY LISTS

- We allow the size to change, but we don't copy over elements every time a new element doesn't fit
- Use the idea of doubling the size of the array to get an average creation time of  $O(n)$  for length  $n$  array

- **Declare/Initialize**

```
ArrayList<String> words = new ArrayList<String>();
```

- **add:**

```
words.add("welcome");  
words.add("spring");  
words.add("break");
```

# ARRAY LISTS

- We allow the size to change, but we don't copy over elements every time a new element doesn't fit
- Use the idea of doubling the size of the array to get an average creation time of  $O(n)$  for length  $n$  array

- **Declare/Initialize**

```
ArrayList<String> words = new ArrayList<String>();
```

- **add:**

```
words.add("welcome");  
words.add("spring");  
words.add("break");
```

- **get:**

```
String w = words.get(2);
```

- **set:**

```
words.set(0, "almost");
```

- **size:**

```
int size = words.size();
```



# MAR 3 OUTLINE

- Queues (theory and implementation)
- Review arrays and ArrayLists
- **Review Nodes and Linked Lists**
- Practice Problems

# NODE OBJECTS

*Match the constructor to the type of list that would contain such Nodes*

```
public Node(E initData) {  
    data = initData;  
    next = null;  
}
```

```
public Node(E initData) {  
    data = initData;  
    next = null;  
    prev = null;  
}
```

- Doubly linked list

- Singly linked list

# NODE OBJECTS

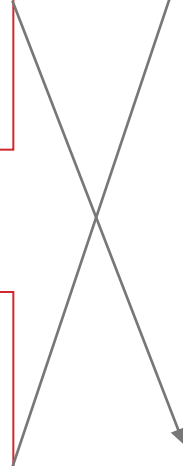
*Match the constructor to the type of list that would contain such Nodes*

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    data = initData;  
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    prev = null;  
}
```

• Doubly linked list

• Singly linked list



# LINKED LISTS

*Match the constructor to the appropriate type(s) of list.*

```
public LinkedList() {  
    head = null;  
    tail = null;  
}
```

```
public LinkedList() {  
    header = new Node(null);  
    trailer = new Node(null);  
    header.setNext(trailer);  
    trailer.setNext(header);  
}
```

```
public LinkedList() {  
    head = null;  
}
```

- Singly linked list
- Singly linked list with tail pointer
- Doubly linked list
- Doubly linked list with tail pointer
- Doubly linked with sentinels

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    trailer.setNext(header);  
}
```

```
public LinkedList() {  
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- Singly linked list
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- Doubly linked with sentinels

# LINKED LISTS

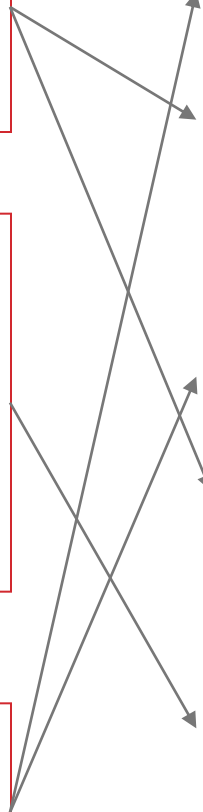
*Match the constructor to the appropriate type(s) of list.*

```
public LinkedList() {  
    head = null;  
    tail = null;  
}
```

```
public LinkedList() {  
    header = new Node(null);  
    trailer = new Node(null);  
    header.setNext(trailer);  
    trailer.setNext(header);  
}
```

```
public LinkedList() {  
    head = null;  
}
```

- Singly linked list
- Singly linked list with tail pointer
- Doubly linked list
- Doubly linked list with tail pointer
- Doubly linked with sentinels



*What is the issue with  
the following code?  
What is printed?*

```
public class LinkedListTest {  
  
    private Node head;  
  
    public LinkedListTest() {  
        head = null;  
    }  
  
    public void addFirst(String newName) {  
        Node newNode = new Node(newName);  
        head = newNode;  
    }  
  
    public static void main(String[] args) {  
        LinkedListTest courses = new LinkedListTest();  
        courses.addFirst("106");  
        courses.addFirst("107");  
        System.out.println(courses);  
    }  
  
    public String toString() {  
        StringBuilder sb = new StringBuilder();  
        Node curr = head;  
        while (curr != null) {  
            sb.append(curr.getName());  
            curr = curr.next();  
        }  
        return sb.toString();  
    }  
}
```

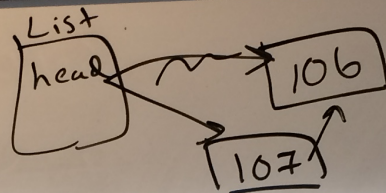
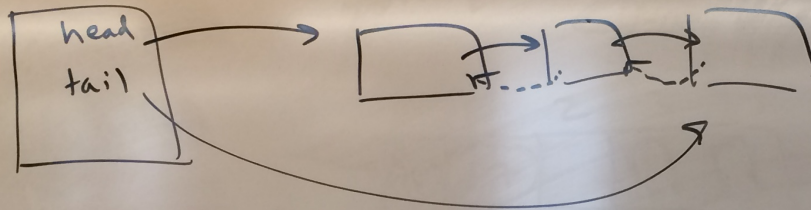


*What is the issue with  
the following code?  
What is printed?*

Only "107"!

```
public class LinkedListTest {  
  
    private Node head;  
  
    public LinkedListTest() {  
        head = null;  
    }  
  
    public void addFirst(String newName) {  
        Node newNode = new Node(newName);  
        head = newNode;  
    }  
  
    public static void main(String[] args) {  
        LinkedListTest courses = new LinkedListTest();  
        courses.addFirst("106");  
        courses.addFirst("107");  
        System.out.println(courses);  
    }  
  
    public String toString() {  
        StringBuilder sb = new StringBuilder();  
        Node curr = head;  
        while (curr != null) {  
            sb.append(curr.getName());  
            curr = curr.next();  
        }  
        return sb.toString();  
    }  
}
```

(1)

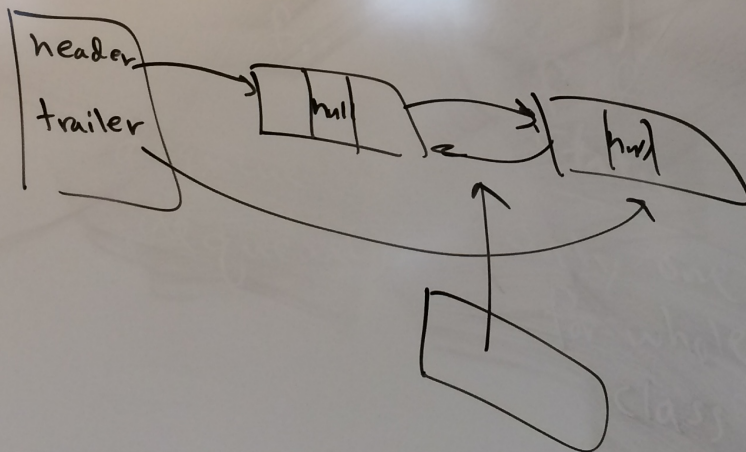


$\text{new Node.next} = \text{head};$

if size is 0:

after first element inserted.

(2)



$\text{Node curr} = \text{header.next()};$

# MAR 3 OUTLINE

- Queues (theory and implementation)
- Review arrays and ArrayLists
- Review Nodes and Linked Lists
- **Practice Problems**

# **WORK WITH A PARTNER!**

- **Question 1: focus on understanding the code and thinking about what type of loops to use (don't rewrite the code now)**
- **Question 2: skip (about Queues)**



# QUESTION 1

① min House : for runtime:  $O(n^2)$   
get Price Index: while runtime:  $O(n)$

while  
inside  
LL

for (House h : list) {  
    h.compareTo(min House)  
}

iterator next

array[i]  $\rightarrow O(1)$   
list.get(i)  
     $\rightarrow O(n)$

```
public class House implements Comparable<House> {  
  
    private int price;  
  
    public House(int initPrice) {  
        price = initPrice;  
    }  
  
    public int getPrice() {  
        return price;  
    }  
}
```

```
public class House implements Comparable<House> {
```

```
    private int price;
```

```
    public House(int initPrice) {  
        price = initPrice;  
    }
```

```
    public int getPrice() {  
        return price;  
    }
```

```
    public int compareTo(House other) {  
        return price - other.getPrice();  
    }
```

```
public class House implements Comparable<House> {

    private int price;

    public House(int initPrice) {
        price = initPrice;
    }

    public int getPrice() {
        return price;
    }

    public int compareTo(House other) {
        return price - other.getPrice();
    }

    public static void main(String[] args) {
        House h1 = new House(20);
        House h2 = new House(30);

        if (h1.compareTo(h2) < 0) {
            System.out.println("House 2 is greater");
        } else if (h1.compareTo(h2) > 0) {
            System.out.println("House 1 is greater");
        } else {
            System.out.println("they are equal");
        }
    }
}
```



# QUESTION 3

There are many ways to do this! Two are shown below. What are the runtimes of these two algorithms?

```
// LL origList contains an even number of elements
// LL list1 and LL list2 are empty

1) boolean flag = true;
   for (String elem : origList) {
       if (flag) {
           list1.add(elem);
           flag = false;
       } else {
           list2.add(elem);
           flag = true;
       }
   }

2) for (int i=0; i < origList.size(); i+=2) {
    list1.add(origList.get(i));
    list2.add(origList.get(i+1));
}
```

# QUESTION 4

- a) `get` & `set` (only public methods)
- b) Could be a `LinkedList` or an `ArrayList`
- c) Less efficient than an `ArrayList`, same efficiency as a `LinkedList`
- d) No! the details are *abstracted* away

## Visualization:

Assume that we have stack A with elements 5, 11, -1, 3 (where 3 is on the top). What happens when we call `get(2)`? Draw the stacks A and B and see what happens.

# QUESTION 5

**a) Pseudocode:**

`tail = tail.prev`

`tail.next = null`

**b) Hints: there are 6 pointers that need to be rearranged (2 for A, 2 for B, and head/tail)**

**d) See notes from Lecture 11**