## Central Limit Theorem example

The central limit theorem (CLT) states that if $X_{1}, X_{2}, \cdots, X_{n}$ are drawn from a population with mean $\mu$, finite variance $\sigma^{2}$, and sample mean $\bar{X}_{n}$, then

$$
Z=\lim _{n \rightarrow \infty} \sqrt{n}\left(\frac{\bar{X}_{n}-\mu}{\sigma}\right)
$$

is a standard normal distribution. The central limit theorem can be used to estimate test statistics, which can be used to compute p-values. The two key assumptions are that the variance is finite and the sample size is large.

In the following example we flip a coin 80 times, and observe 54 heads. The question is: is this coin unfair?

1. Coin tosses can be thought of as samples from the Bernoulli distribution, where Tails=0 and Heads=1. We can write this as $X \sim \operatorname{Ber}(p)$, where $p$ is the probability that $X=1$. For a fair coin, what is the expected value of $X$, i.e. $E[X]$ ?
2. For a fair coin, what is the variance of $X$, i.e. $\operatorname{Var}(X)$ ? Use the fact that $\operatorname{Var}(X)=E\left[(X-\mu)^{2}\right]$, where $\mu=E[X]$.
3. For the example above ( 54 heads out of 80 tosses), what is the sample mean $\bar{X}_{n}$ ?
4. Putting this all together and using the CLT, what is the test statistic? (use a calculator)
5. Sketch out the position of the test statistic on a standard normal distribution. Shade the area(s) representing the corresponding p -value.
6. Based on your diagram, do you think this coin is fair?
