Naive Bayes
Say we have two tests for a specific disease. Each test (features $f_{1}, f_{2}$ ) can come back either positive "pos" or negative "neg", and the true underlying condition of the patient is represented by $y$ ( $y=1$ is "healthy" and $y=2$ is "disease"). We observe this training data where $n=7$ and $p=2$ :

| $\boldsymbol{x}$ | $f_{1}$ | $f_{2}$ | $y$ |
| :---: | :---: | :---: | :---: |
| $\boldsymbol{x}_{1}$ | pos | neg | 1 |
| $\boldsymbol{x}_{2}$ | pos | pos | 2 |
| $\boldsymbol{x}_{3}$ | pos | neg | 2 |
| $\boldsymbol{x}_{4}$ | neg | neg | 1 |
| $\boldsymbol{x}_{5}$ | pos | neg | 2 |
| $\boldsymbol{x}_{6}$ | neg | neg | 1 |
| $\boldsymbol{x}_{7}$ | neg | pos | 2 |

1. To estimate the probability $p(y=k)$, for $k=1,2, \cdots, K$, we will use the formula:

$$
\theta_{k}=\frac{N_{k}+1}{n+K}
$$

where $N_{k}$ is the count ("Number") of data points where $y=k$. Compute $\theta_{1}$ and $\theta_{2}$. What would $\theta_{1}$ and $\theta_{2}$ be if we in fact had no training data?
2. To estimate the probabilities $p\left(x_{j}=v \mid y=k\right)$ for all features $j$, values $v$, and class label $k$, we will use the formula:

$$
\theta_{k, j, v}=\frac{N_{k, j, v}+1}{N_{k}+\left|f_{j}\right|}
$$

where $N_{k, j, v}$ is the count of data points where $y=k$ and $x_{j}=v$, and $\left|f_{j}\right|$ is the number of possible values that $f_{j}$ (feature $j$ ) can take on. Fill in the following tables with these $\theta$ values.

| $y=1$ | pos | neg |
| :---: | :---: | :---: |
| $f_{1}$ |  |  |
|  |  |  |
| $f_{2}$ |  |  |
|  |  |  |


| $y=2$ | pos | neg |
| :---: | :---: | :---: |
| $f_{1}$ |  |  |
|  |  |  |
| $f_{2}$ |  |  |
|  |  |  |

