

# CS 360: Machine Learning

Prof. Sara Mathieson

Fall 2020



# Admin

- **Lab 6 due Wed**
  - Grace period until Thurs night
  - Help hours Sun/Mon/Tues
- Welcome prospective students!

# Outline for October 23

- Evaluation Metrics
  - Confusion matrices revisited
  - ROC curves
  - Relationship to probabilistic methods
- Recap Random forests
- AdaBoost

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## For now: assume binary classification task

- Transactions that indicate credit card fraud
- Detecting which scans show tumors
- Prenatal test for Down's Syndrome
- Finding genes under natural selection
- Finding regions of the genome with high recombination rate ("hotspots")

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In all these examples, we are trying to find unusual items (“needle in a haystack”) -- we call these *positives*

# Goals of Evaluation

- Think about what metrics are important for the problem at hand
- Compare different methods on the same problem
- Common set of tools that other researchers/users can understand

# Precision and Recall

- Precision: of all the “flagged” examples, which ones are actually relevant (i.e. positive)?
- Recall: of all the relevant results, which ones did I actually return?

# Precision and Recall

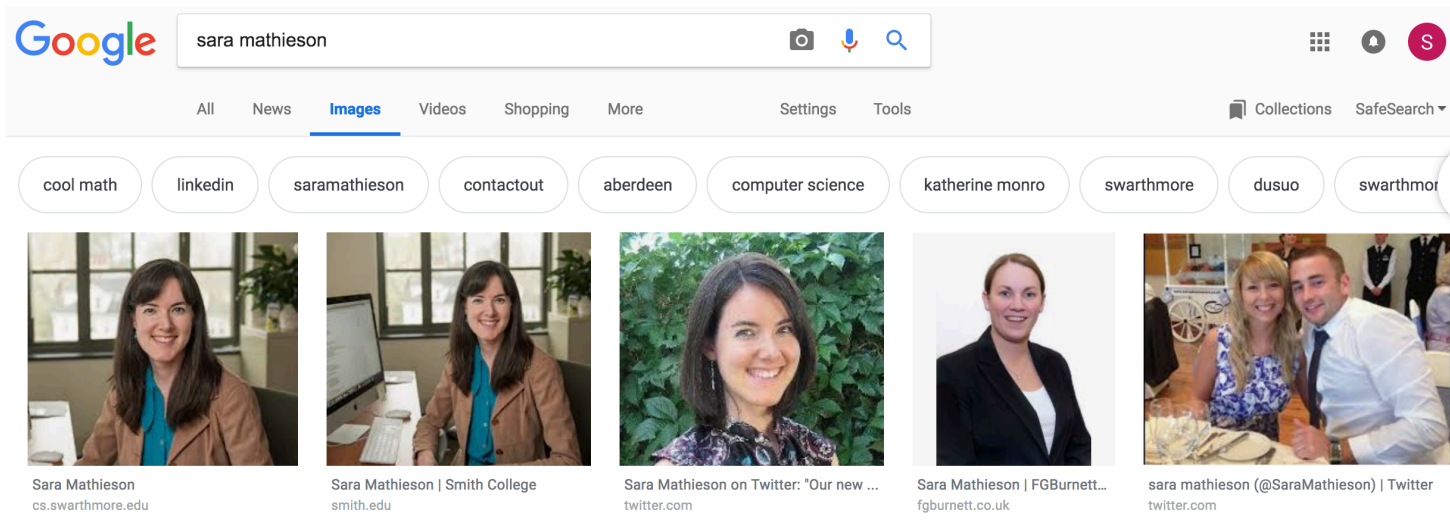
- Precision: of all the “flagged” examples, which ones are actually relevant (i.e. positive)?

(Purity)

- Recall: of all the relevant results, which ones did I actually return?

(Completeness)

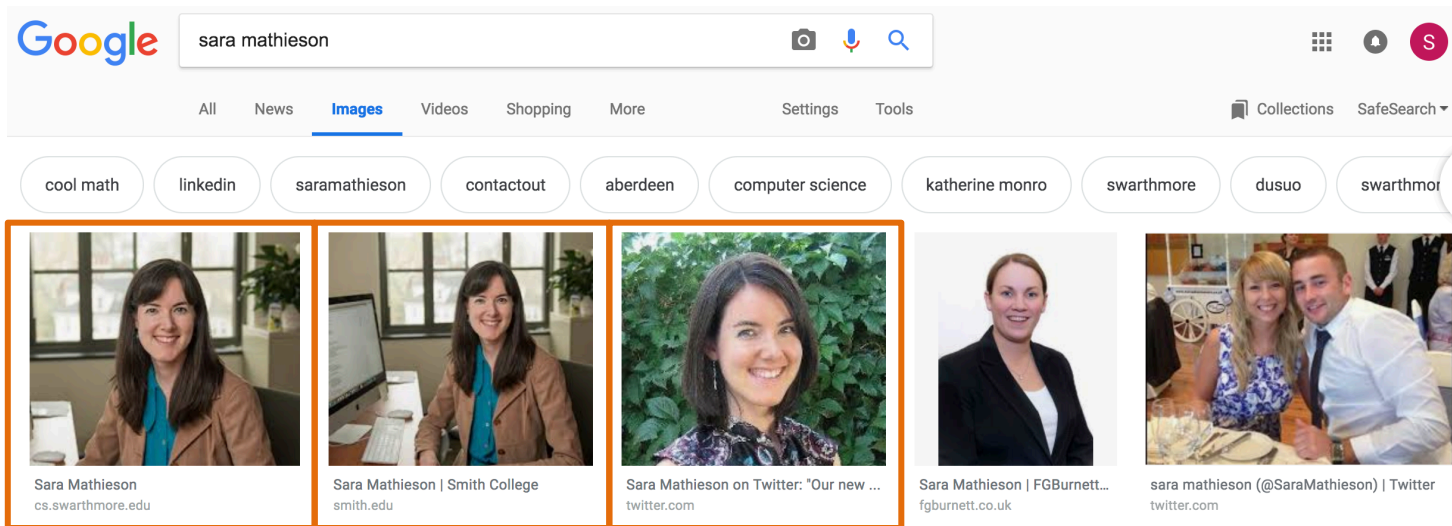
# Precision and Recall



$P=6$  (number of images that are actually me)

- Precision?
- Recall?

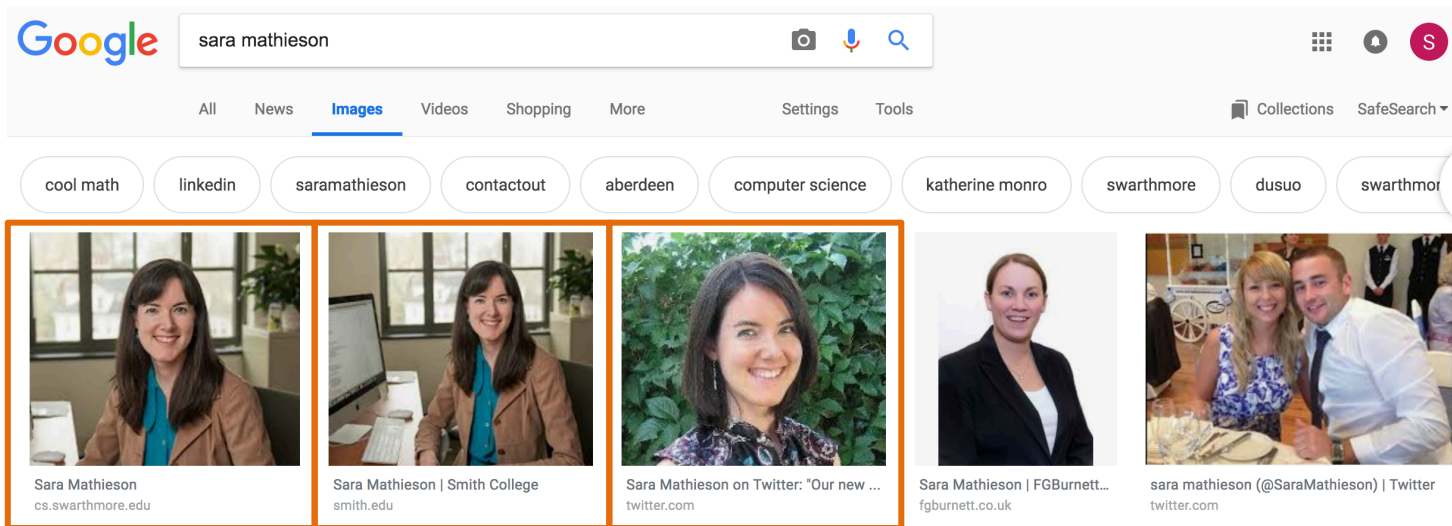
# Precision and Recall



$P=6$  (number of images that are actually me)

- Precision =  $TP/(FP+TP) = 3/5$
- Recall?

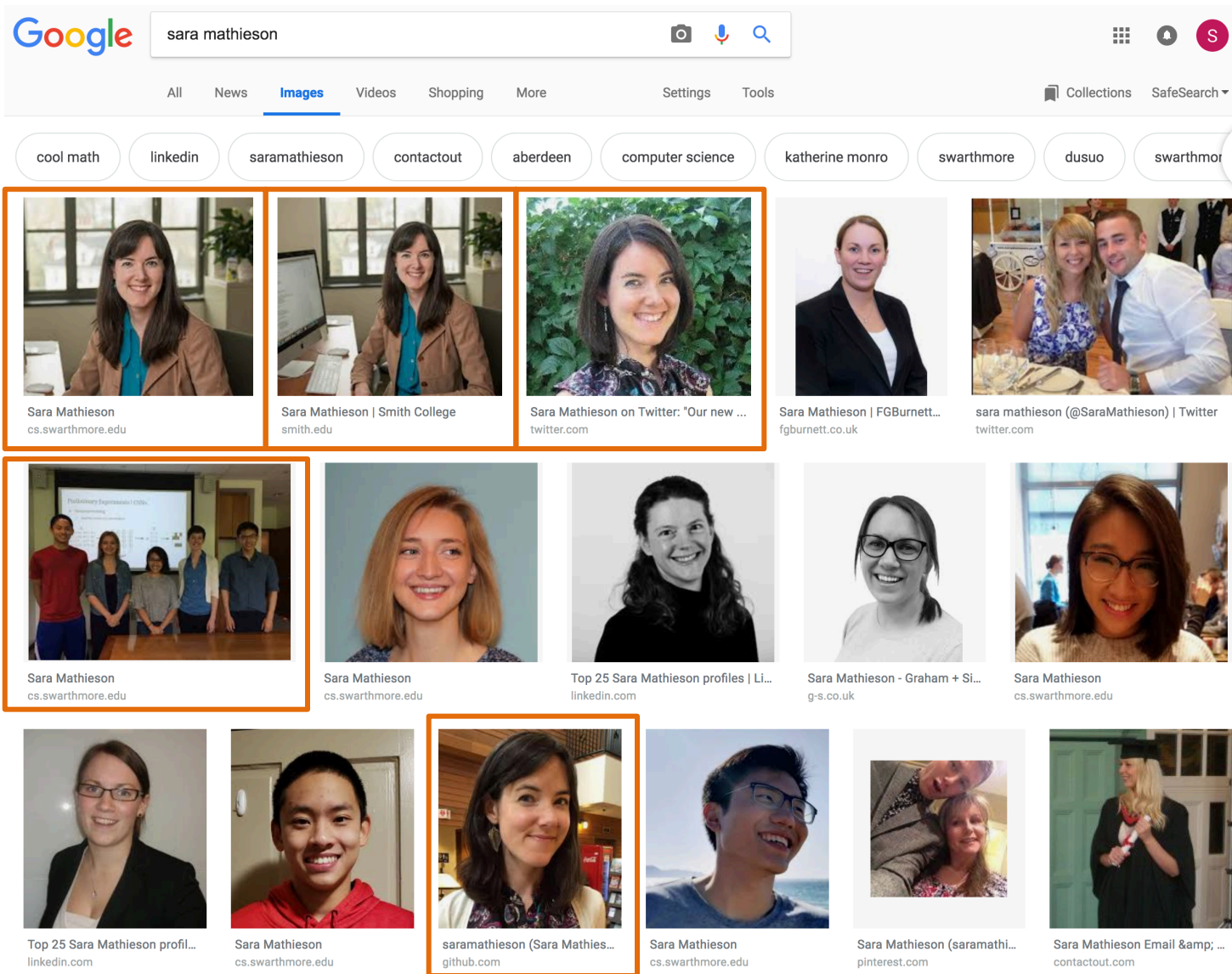
# Precision and Recall



$P=6$  (number of images that are actually me)

- Precision =  $TP/(FP+TP) = 3/5$
- Recall =  $TP/(FN+TP) = 3/6$

# Precision and Recall



$P=6$  (number of images that are actually me)

- Precision =  $5/16$
- Recall =  $5/6$

# Revisit Confusion Matrices

		Predicted class	
		Negative	Positive
True class	Negative	True negative (TN)	False positive (FP)
	Positive	False negative (FN)	True positive (TP)

# Revisit Confusion Matrices

Predicted class

Negative

Positive

Negative

True negative  
(TN)

False positive  
(FP)  
“false alarm”

N (total number of true negatives)

True  
class

Positive

False negative  
(FN)  
“miss”

True positive  
(TP)

P (total number of true positives)

N\* (what we said  
was negative)

P\* (what we said was  
positive “flagged”)

# Revisit Confusion Matrices

		Predicted class		
		Negative	Positive	
True class	Negative	True negative (TN) ✓	False positive (FP) "false alarm" ✗	N
	Positive	False negative (FN) "miss" ✗	True positive (TP) ✓	P
		N*	p*	

# Revisit Confusion Matrices

		Predicted class	
		Negative	Positive
True class	Negative	True negative (TN)	False positive (FP) "false alarm"
	Positive	False negative (FN) "miss"	True positive (TP)
		N*	p*

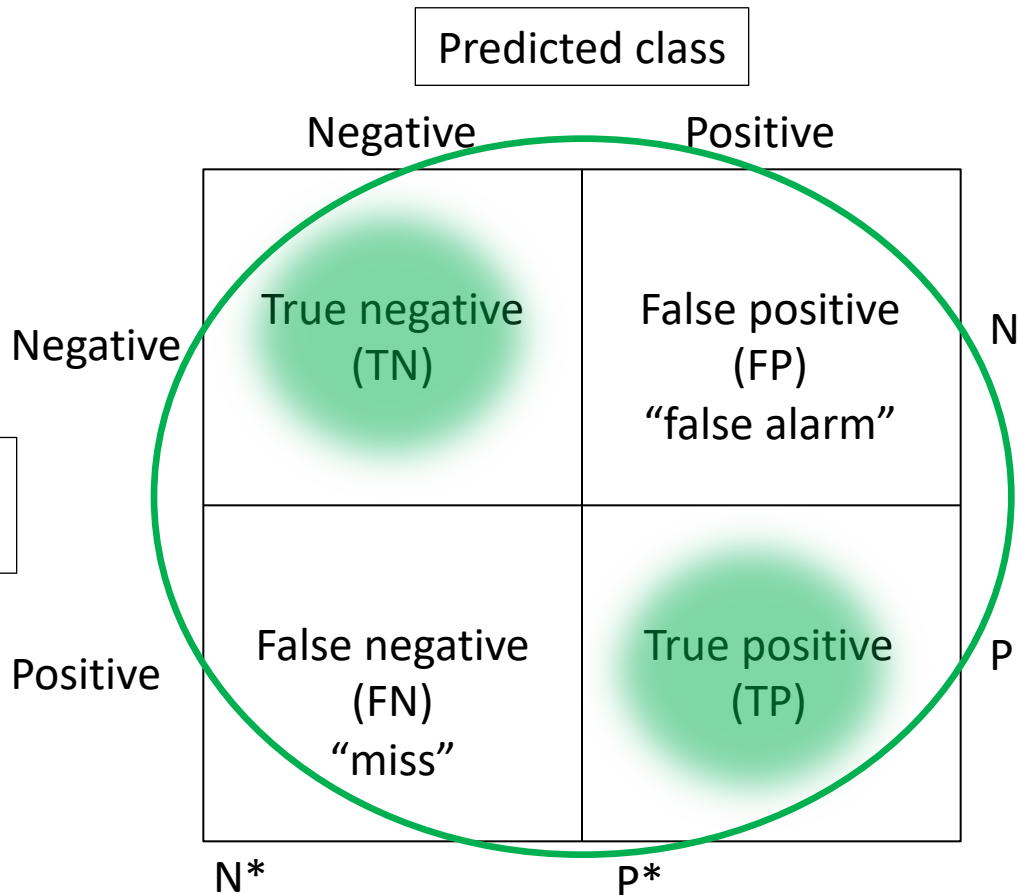
A red circle highlights the False Positive (FP) and False Negative (FN) cells, representing the total error. The total number of negative samples is labeled 'N' on the right, and the total number of positive samples is labeled 'P' on the right. The total number of negative samples predicted is 'N\*' at the bottom left, and the total number of positive samples predicted is 'p\*' at the bottom right.

Error:

$$(FN+FP)/(TN+FP+FN+TP)$$

$$= (FN+FP)/(N+P)$$

# Revisit Confusion Matrices



Accuracy = 1-Error:

$$(TN+TP)/(TN+FP+FN+TP)$$

$$= (TN+TP)/(N+P)$$

# Revisit Confusion Matrices

		Predicted class		
		Negative	Positive	
True class	Negative	True negative (TN)	False positive (FP) "false alarm"	N
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		N*	P*	

Precision:

$$TP/(FP+TP) = TP/P^*$$

# Revisit Confusion Matrices

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Recall  
(True Positive Rate):

$$TP/(FN+TP) = TP/P$$

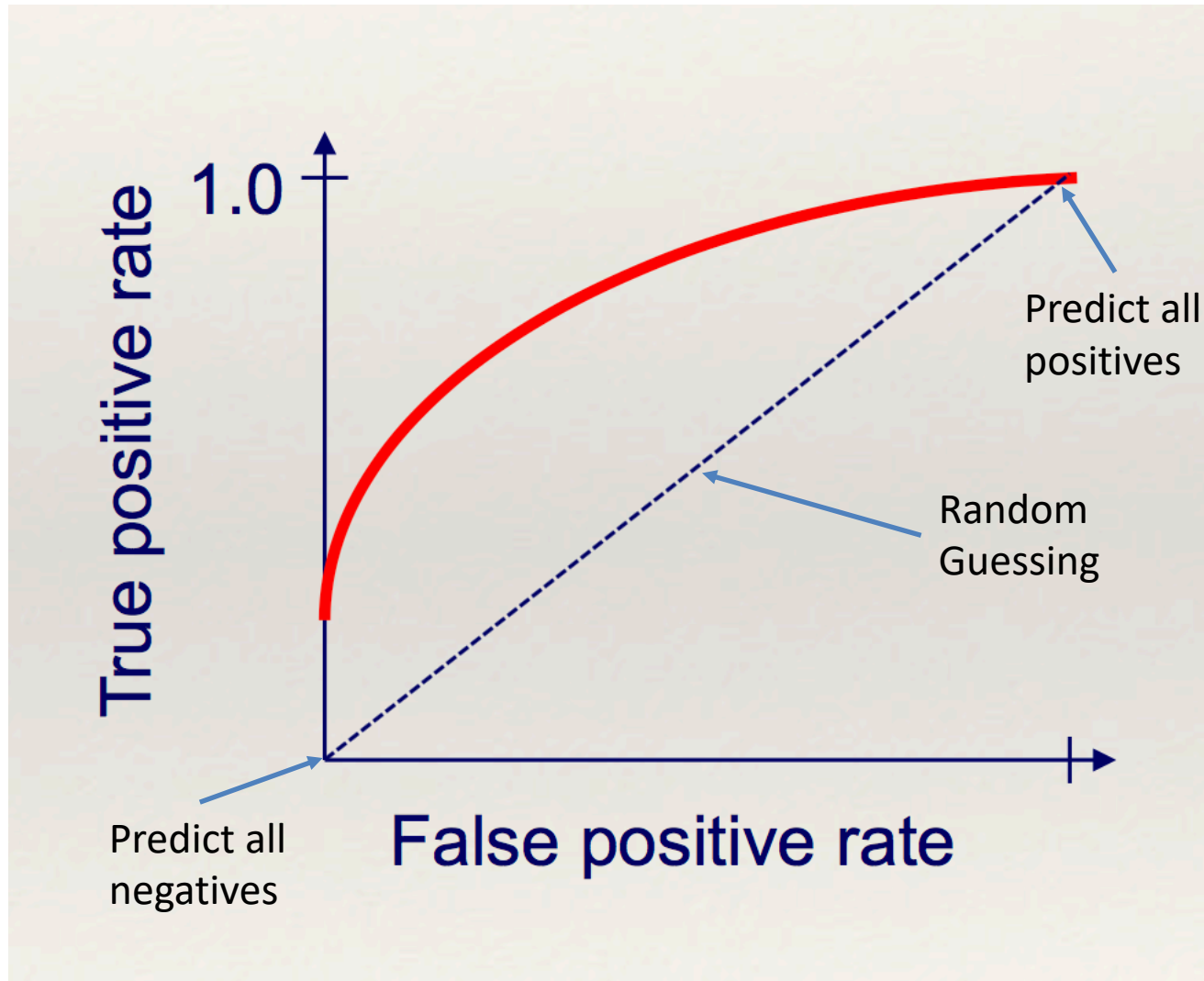
# Revisit Confusion Matrices

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		N*	p*	

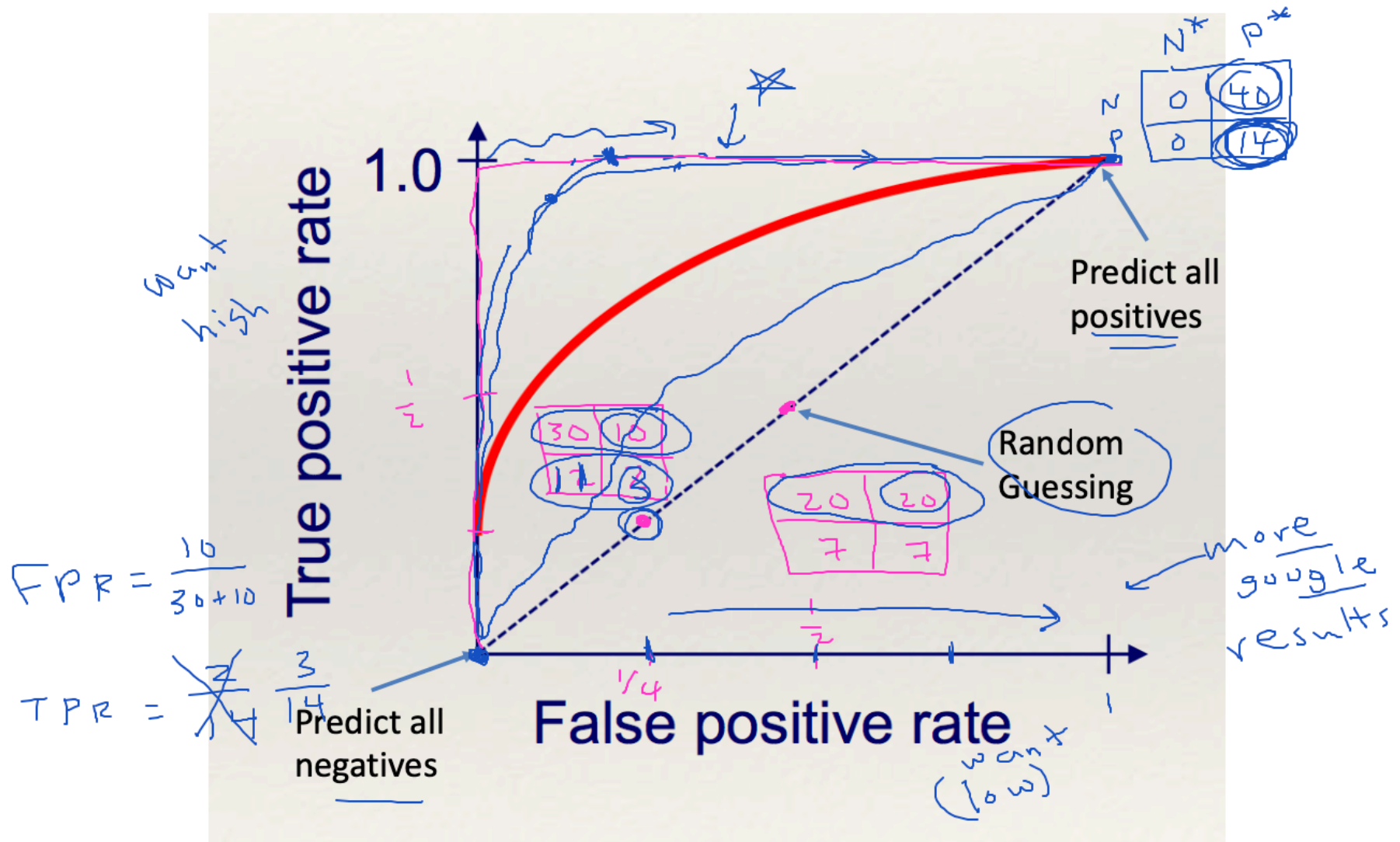
False Positive Rate:

$$FP/(TN+FP) = FP/N$$

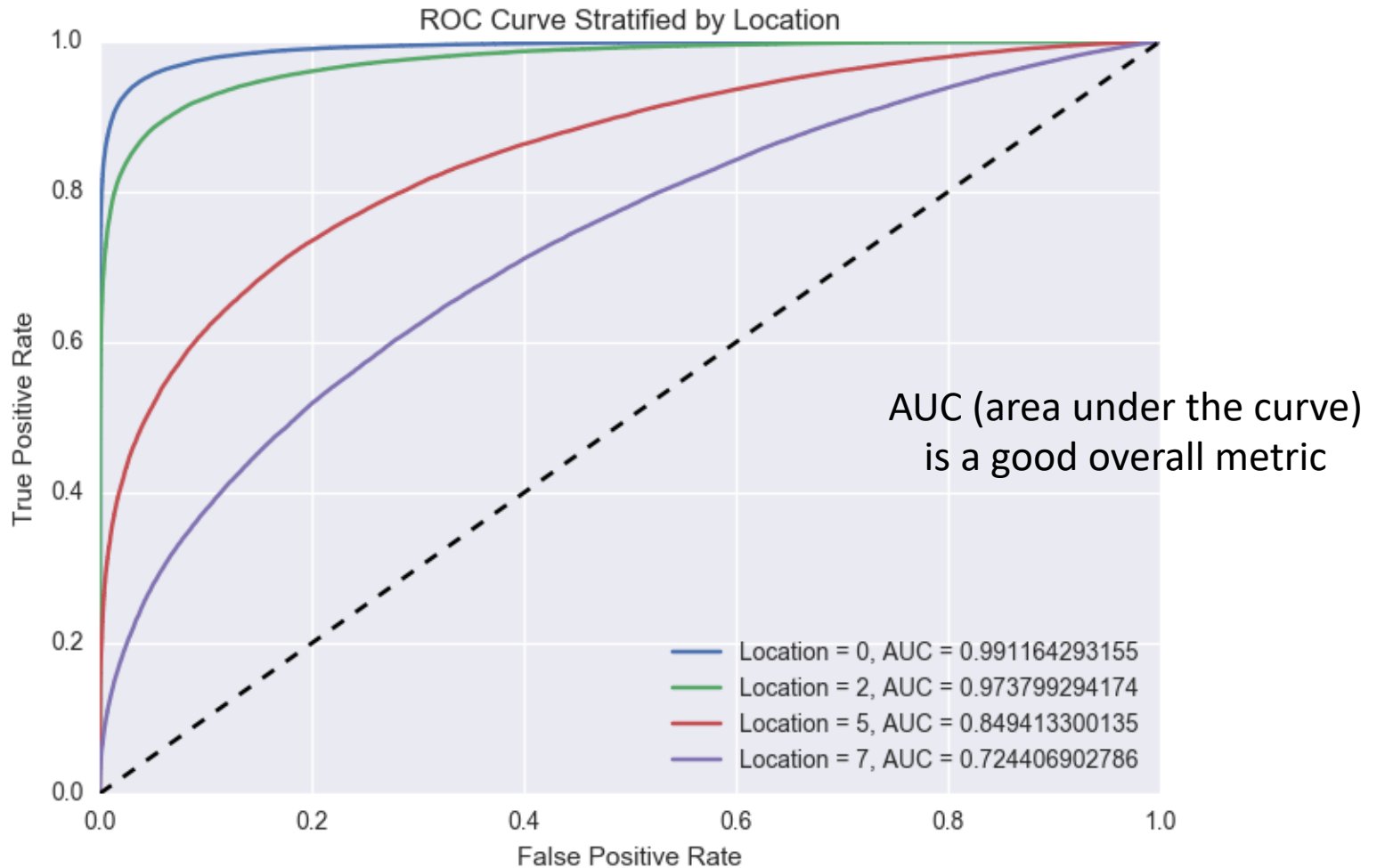
# ROC curve (Receiver Operating Characteristic)



# ROC curve (Receiver Operating Characteristic)



# ROC curve example: comparing methods



Example of a ROC curve from my research  
Chan, Perrone, Spence, Jenkins, Mathieson, Song

# How to get a ROC curve for probabilistic methods?

- Usually we use 0.5 as a threshold for binary classification
- Vary the threshold! (i.e. choose 0.25)
  - $P(y=1 \mid x) > 0.25 \quad \Rightarrow$  classify as 1 (positive)
  - $P(y=1 \mid x) \leq 0.25 \quad \Rightarrow$  classify as 0 (negative)

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$p(y=1 \mid x) > 0.9 \Rightarrow$  few positives

# Handout 12

①

pred

77	3	→ N = 80
13	7	→ P = 20

true

②

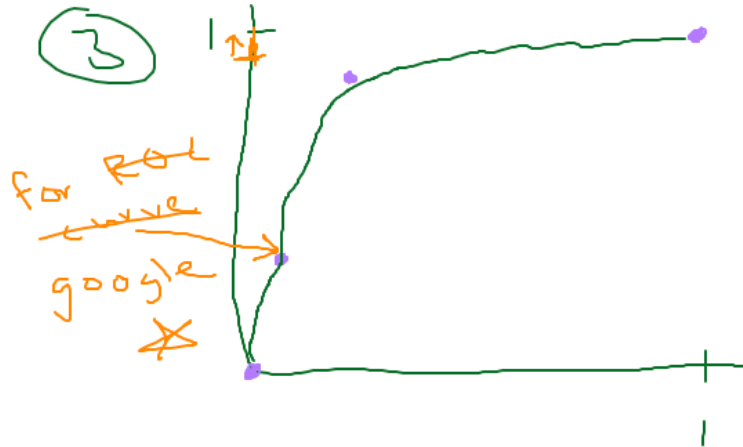
precision

$$\frac{7}{3+7} = 0.7$$

recall = TPR

$$= \frac{7}{20} = \underline{0.35}$$

③



$$FPR = \frac{3}{80} = 0.04$$

④

68	12
2	18

$$FPR = \frac{12}{80} = 0.15$$

$$TPR = \frac{18}{20} = \underline{0.9}$$

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# Random Forests

- Idea: choose a different **subset of features** for every classifier  $t$
- Typically use *decision stumps* (depth 1)
- Goal: decorrelate models
- In practice: choose  **$\sqrt{p}$**  features
  - Without replacement for each model
  - Every model: data points and features chosen independently

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# Boosting Idea

## TRAIN

- Start with all example equally weighted
- For  $T$  iterations:
  - Learn weak classifier using weighted examples
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## TEST

- Get predictions from all weak classifiers
- Vote based on how well each classifier did during training

# AdaBoost (adaptive boosting)

start:  $w_i^{(1)} = \frac{1}{n}$  for all training examples

for  $t = 1 \dots T$   $\leftarrow$  # classifiers

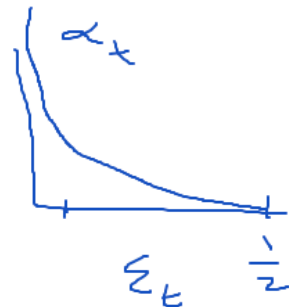
(a) fit classifier to weighted examples  
 $\Rightarrow h^{(t)}(\vec{x})$

(b) weighted class. error

want low  $\rightarrow \epsilon_t = \sum_{i=1}^n w_i^{(t)} \mathbb{I}(\underbrace{y_i}_{\text{true}} \neq \underbrace{h^{(t)}(\vec{x}_i)}_{\text{pred}})$   
error

(c) compute score  $\leftarrow$  want high

$$\alpha_t = \frac{1}{2} \log \left( \frac{1 - \epsilon_t}{\epsilon_t} \right)$$




# AdaBoost (adaptive boosting)

model

test

$$\tilde{h}(x) = \text{sign} \left( \sum_{t=1}^T \underbrace{\alpha_t}_{\text{model score}} \underbrace{h^{(t)}(x)}_{\{-1, 1\}} \right)$$

↑



A horizontal line represents a 1D feature space. A vertical tick mark on the line is labeled with a circle containing the letter 'D'. To the left of the tick mark, there are two arrows pointing left, and to the right, there are two arrows pointing right, indicating a threshold or decision boundary.

$$\alpha_t = \frac{1}{2} \log \left( \frac{1 - \epsilon_t}{\epsilon_t} \right)$$

