Convolutional Neural Networks *(find and work with a partner)*

1. *Activation Functions.* So far we have seen three different activation functions:

   - Sigmoid: $\sigma(x) = \frac{1}{1+e^{-x}}$
   - Hyperbolic Tangent: $\tanh(x) = \frac{e^x - e^{-x}}{e^x + e^{-x}}$
   - ReLU: $f(x) = \max(0, x)$

   If the input has value exactly 0, what is the activation of each function above?

2. *Neural Network Architectures.* Say we have a fully connected neural network with 3 layers. The input data has shape $(n, p) = (100, 3)$, the first and second hidden layers each have 4 units (i.e. $p_1 = p_2 = 4$), and we have one output. If we use biases for all 3 layers, how many total parameters do we need to optimize?

3. *Cross-Entropy Loss.* We define cross-entropy between two discrete probability distributions p and q (with the same set of inputs $\mathcal{X}$) as

$$H(p, q) = -\sum_{x \in \mathcal{X}} p(x) \log_2 q(x).$$

   When using cross-entropy as a classification loss function between the true one-hot class label vector y and the output probabilities for each class $\hat{y}$, we have

$$H(y, \hat{y}) = -\sum_{k=1}^{K} y_k \log_2 \hat{y}_k,$$

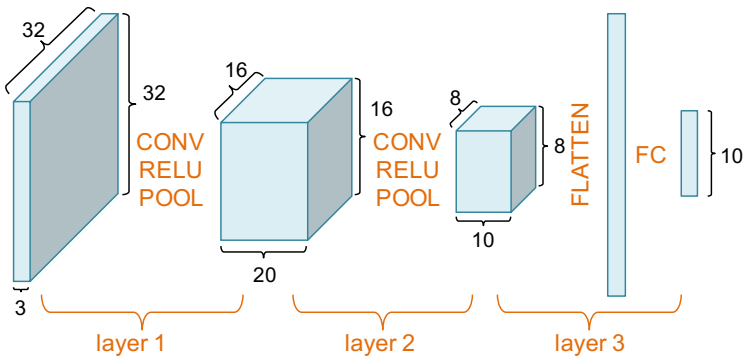
   where K is the number of classes. Say we have $K = 3$. For a given data point $\vec{x}$, say the true label is class 2, and our neural network produced the probabilities $\hat{y} = [\frac{1}{4}, \frac{1}{2}, \frac{1}{4}]$. What is the cross-entropy loss for this data point?

   If instead our method produced probabilities $\hat{y} = [\frac{3}{8}, \frac{1}{8}, \frac{1}{2}]$, what is the loss for this data point?

   Note: for multiple examples, we usually use the loss function:

$$J(\text{all weights}) = \frac{1}{n} \sum_{i=1}^{n} H(y_i, \hat{y}_i)$$

4. Say we use a 3-layer CNN with architecture shown below. Our inputs have shape $(32 \times 32 \times 3)$.



- In the CONV step of the first layer, we use 20 filters, each (5×5) in width and height, but all the way through the depth. We use a stride of 1 in each dimension, and use padding = "SAME" to make sure the input and output width/height are the same. After CONV, we apply a RELU non-linearity, then apply POOL using a max-pooling strategy with (2×2) filters and stride 2 in width/height. This reduces the width and height by a factor of 2.
- In the second layer we use 10 filters, each (3×3) in width and height, but all the way through the depth. The stride and padding follow the same procedure as the first layer. ReLU and pooling also follow the same strategy.
- Finally, we flatten the volume in preparation for the full connected layer. The FC layer transforms the flattened volume into scores for 10 classes.

(a) Which steps (i.e. CONV, RELU, POOL, FLATTEN, FC) require parameter learning through gradient descent? Which steps don't?

(b) How many parameters do we need to learn for the first layer? What if we also included a bias for each filter?

(c) How many parameters do we need to learn for the second layer? What if we also included a bias for each filter?

(d) How many parameters do we need to learn for the third (FC) layer? What if we also included a bias for each class?

(e) Assuming we keep the biases for each layer, how many parameters total do we need to learn?