## Working with Likelihoods

(find and work with a partner)

1. Bernoulli Random Variable. Say we flip a weighted coin n times, and each time the probability of heads (1) is p, so the probability of tails (0) is (1 - p). Let  $y_i$  be the outcome of flip i. For example, if n = 10, we might observe these values:

$$\boldsymbol{y} = [0, 0, 1, 1, 0, 1, 0, 1, 0, 0]$$

In this case, the *likelihood* of p given this observed data is  $L(p) = p^4(1-p)^6$ , since we observe four 1's and six 0's. In general, we can write the likelihood as

$$L(p) = \prod_{i=1}^{n} p^{y_i} (1-p)^{1-y_i},$$

so that for each  $y_i$ , only one of  $y_i$  and  $(1 - y_i)$  will be non-zero and contribute to the product. Note that  $L(p | \mathbf{y})$  is a more proper way of writing this (i.e. given the data), but we often omit this conditional part.

(a) What is the log likelihood  $\ell(p)$  for this setup? Simplify as much as possible.

(b) Our goal is to *maximize* the log likelihood. Take the derivative with respect to p and set it equal to 0. Solve for p – this becomes our MLE (maximum likelihood estimator),  $\hat{p}$ .

(c) For our concrete example above with n = 10, what is the MLE  $\hat{p}$ ? Does this match your intuition?