Simple Linear Regression *(find and work with a partner)*

In our linear regression setup, we are given a matrix $\boldsymbol{X}$ which consists of n data points, each with p features (also called predictors or explanatory variables), plus a "1" in the first column. Each data point $\boldsymbol{x}_i$ has an associated response variable y_i. Together these form $\boldsymbol{y}$. In multiple linear regression, $p > 1$ and our model can be described by a vector of coefficients, $\boldsymbol{w}$. In matrix-vector form, we have

$$\boldsymbol{y} = \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{bmatrix}, \qquad \boldsymbol{X} = \begin{bmatrix} 1 & — & \boldsymbol{x}_1 & — \\ 1 & — & \boldsymbol{x}_2 & — \\ & & \vdots & \\ 1 & — & \boldsymbol{x}_n & — \end{bmatrix}, \qquad \boldsymbol{w} = \begin{bmatrix} w_0 \\ w_1 \\ w_2 \\ \vdots \\ w_p \end{bmatrix}$$

where each example $\boldsymbol{x}_i = \begin{bmatrix} x_{i1}, \dots, x_{ip} \end{bmatrix}$. Our linear regression model is:

$$h_{\boldsymbol{w}}(\boldsymbol{x}) = \boldsymbol{w}^T \boldsymbol{x}$$

Beginning with the simple linear regression case ($p = 1$), we have:

$$h_{\boldsymbol{w}}(x) = w_0 + w_1 x$$

1. *Toy example.* Let $n = 2$ and $p = 1$, with the following data (we will omit the first column of 1's in simple linear regression):

$$\boldsymbol{y} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \qquad \boldsymbol{X} = \begin{bmatrix} 1 \\ 0 \end{bmatrix},$$

   (a) Plot these two points – what should $\hat{w}_0$ and $\hat{w}_1$ be?

   (b) This week we derived the solution for simple linear regression:

$$\hat{w}_1 = \frac{\mathrm{Cov}(\boldsymbol{x}, \boldsymbol{y})}{\mathrm{Var}(\boldsymbol{x})} = \frac{\frac{1}{n}\sum_{i=1}^{n}(x_i - \bar{x})(y_i - \bar{y})}{\frac{1}{n}\sum_{i=1}^{n}(x_i - \bar{x})^2} \qquad \hat{w}_0 = \bar{y} - \hat{w}_1 \bar{x}$$

   where $\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i$ and $\bar{y} = \frac{1}{n}\sum_{i=1}^{n} y_i$. Use these equations to compute $\hat{w}_0$ and $\hat{w}_1$ and verify your answer to (a).