Viterbi Algorithm for HMMs

1. Let K be the total number of hidden states and B be the number of possible emissions. Let L be the length of the Markov chain (as well as the length of the emitted sequence). How many possible hidden state sequences are there? i.e., how many possible paths through the dynamic programming table are there?

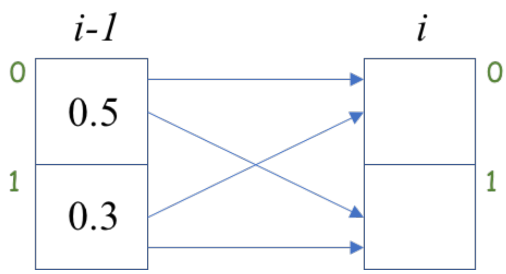
2. What is the runtime of the Viterbi algorithm, in terms of K, B, and L?

3. *Example 1*: The table on the left represent the transition probabilities in a Markov chain with $K = 2$ states (0 and 1). The table on the right represents the emission probabilities for each state of the $B = 3$ possible observation values. Fill in the rest of the transition and emission probabilities.

		End state	
		0	1
Start state	0	0.2	
	1		0.4

		Emitted observation		
		A	B	C
Hidden state	0	0.5		0.4
	1		0.3	0.6

Assuming that the i^{th} observation $X_i = A$, complete one step of the Viterbi algorithm (i.e. label each arrow with its transition probability and then fill in the missing cells). Highlight the arrows we will keep for traceback. Note: state labels are in green below, so $V_0(i-1) = 0.5$ and $V_1(i-1) = 0.3$.



4. Assuming that $X_{i+1} = B$, compute another step of the Viterbi Algorithm. What is the most likely series of hidden states from $i - 1$ to $i + 1$?

5. You'll notice that the numbers in your table are becoming smaller and smaller (because you are always multiplying by probabilities that are < 1). Eventually this will be a problem because the numbers will be too small to be represented by a computer. Suggest how to get around this problem.