

**Naive Bayes***(find and work with a partner)*

Say we have two tests for a specific disease. Each test (features  $f_1, f_2$ ) can come back either positive “pos” or negative “neg”, and the true underlying condition of the patient is represented by  $y$  ( $y = 1$  is “healthy” and  $y = 2$  is “disease”). We observe this training data where  $n = 7$  and  $p = 2$ :

$\mathbf{x}$	$f_1$	$f_2$	$y$
$\mathbf{x}_1$	pos	neg	1
$\mathbf{x}_2$	pos	pos	2
$\mathbf{x}_3$	pos	neg	2
$\mathbf{x}_4$	neg	neg	1
$\mathbf{x}_5$	pos	neg	2
$\mathbf{x}_6$	neg	neg	1
$\mathbf{x}_7$	neg	pos	2

1. To estimate the probability  $p(y = k)$ , for  $k = 1, 2, \dots, K$ , we will use the formula:

$$\theta_k = \frac{N_k + 1}{n + K}$$

where  $N_k$  is the count (“Number”) of data points where  $y = k$ . Compute  $\theta_1$  and  $\theta_2$ . What would  $\theta_1$  and  $\theta_2$  be if we in fact had *no* training data?

2. To estimate the probabilities  $p(x_j = v | y = k)$  for all features  $j$ , values  $v$ , and class label  $k$ , we will use the formula:

$$\theta_{k,j,v} = \frac{N_{k,j,v} + 1}{N_k + |f_j|}$$

where  $N_{k,j,v}$  is the count of data points where  $y = k$  and  $x_j = v$ , and  $|f_j|$  is the number of possible values that  $f_j$  (feature  $j$ ) can take on. Fill in the following tables with these  $\theta$  values.

$y = 1$	pos	neg
$f_1$		
$f_2$		

$y = 2$	pos	neg
$f_1$		
$f_2$		