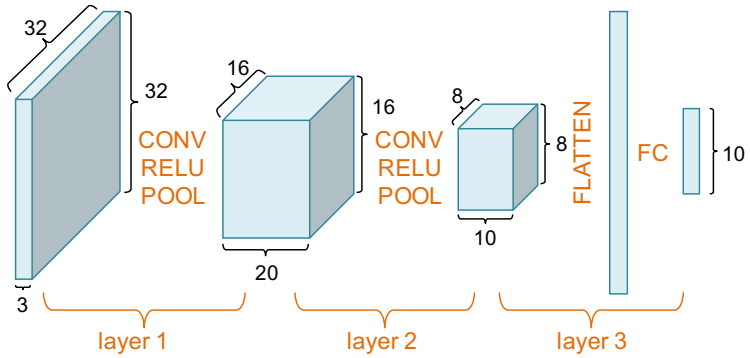
Convolutional Neural Networks

(find and work with a partner)

1. Say we use a 3-layer CNN with architecture shown below. Our inputs have shape $(32 \times 32 \times 3)$.



- In the CONV step of the first layer, we use 20 filters, each (5×5) in width and height, but all the way through the depth. We use a stride of 1 in each dimension, and use padding = "SAME" to make sure the input and output width/height are the same. After CONV, we apply a RELU non-linearity, then apply POOL using a max-pooling strategy with (2×2) filters and stride 2 in width/height. This reduces the width and height by a factor of 2.
- In the second layer we use 10 filters, each (3×3) in width and height, but all the way through the depth. The stride and padding follow the same procedure as the first layer. ReLU and pooling also follow the same strategy.
- Finally, we flatten the volume in preparation for the full connected layer. The FC layer transforms the flattened volume into scores for 10 classes.

(a) Which steps (i.e. CONV, RELU, POOL, FLATTEN, FC) require parameter learning through gradient descent? Which steps don't?

(b) How many parameters do we need to learn for the first layer? What if we also included a bias for each filter?

(c) How many parameters do we need to learn for the second layer? What if we also included a bias for each filter?

(d) How many parameters do we need to learn for the third (FC) layer? What if we also included a bias for each class?

(e) Assuming we keep the biases for each layer, how many parameters total do we need to learn?

(f) If we had instead used a 3-layer FC network for the same input/output with $p_1 = 100$ units in the first hidden layer and $p_2 = 50$ units in the second hidden layer (+ biases for all layers), we would have needed 312,860 parameters (work this out after class). How much of an improvement is the CNN?

2. Say we have an input width of $W = 10$, a filter size $F = 7$, padding of $P = 3$ on each side, and a stride of $S = 3$.

   (a) Using the formula for output size:

   $$\frac{W - F + 2P}{S} + 1$$

   what is the output size for these parameters?

   (b) On the figure below representing the input width, draw the padding and show how your answer above makes sense.

   (c) In this case, the filter performs a cross-correlation on only a subset of the units in the input. Shade in these units on your figure above (i.e. those in the center of the filter).

3. If our input width was $W = 32$ and we used a filter size $F = 5$ with stride $S = 1$, what padding would we need to make the output size the same as our input size?

4. If we use a stride $S > 1$, does it make sense to require the input and output dimension to match? Why or why not?