CS 66: Machine Learning

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SVM Problem Set Notes

SVM Pset

(3) $\max_x \min_y f(x,y) \leq \min_y \max_x f(x,y)$

LHS (RHS)

LHS

let: $\min_y f(x,y) = f_{y_s}(x)$ — y "small"

$\Rightarrow x^* = \operatorname{argmax}_x f_{y_s}(x)$

$f_{y_s}(x^*) \leq f(x^*, y) \quad \forall\, x, y,$ including x^*

RHS

let $\max_x f(x,y) = f_{x_\ell}(y)$ — x "large"

$\Rightarrow y^* = \operatorname{argmin}_y f_{x_\ell}(y)$

$\Rightarrow f_{x_\ell}(y^*) \geq f(x, y^*) \quad \forall\, x, y,$ including y^*

put together

$f_{y_s}(x^*) \leq f(x^*, y^*) \leq f_{x_\ell}(y^*)$

$f_{y_s}(x^*) = \text{LHS}$

$f_{x_\ell}(y^*) = \text{RHS}$

④ $y_i(\vec{w}^* \cdot \vec{x}_i + b) \geq 1$

γ_i

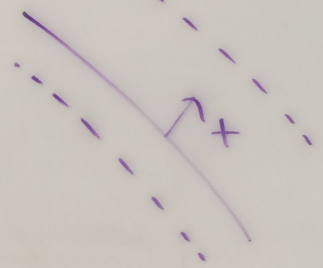
overall / functional margin

$y_i = 1$ $\quad \min_{i : y_i = 1} (\vec{w}^* \cdot \vec{x}_i + b) = \hat{\gamma}$ (positive)

$y_i = -1$ $\quad -\max_{i : y_i = -1} (w^* \cdot \vec{x}_i + b) = \hat{\gamma}$ (negative)

y_i

Set equations equal



⑤ $\vec{w} = a \begin{bmatrix} 1 \\ - \end{bmatrix}$

$\vec{x}_s$

$1 = 1 \cdot \left(a \begin{bmatrix} 1 \\ 2 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 0 \end{bmatrix} + b \right)$

$\vec{x}_1$

$1 = -1 \left(a \begin{bmatrix} 1 \\ 2 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 5 \end{bmatrix} + b \right)$

} solve for a & b

$b^* = \frac{6}{5}$, $a = -\frac{1}{5}$

$\vec{w}^* = \begin{bmatrix} -1/5 \\ -2/5 \end{bmatrix}$

$$\vec{w} = \sum_{i=1}^{n} y_i \vec{x}_i \alpha_i$$

$$\sum \alpha_i y_i = 0$$

all α's greater than 0!

$\alpha_1 = \frac{1}{20}$ -1

$\alpha_2 = 0$

$\alpha_3 = \frac{1}{20}$ -1

$\alpha_4 = 0$

$\alpha_5 = \frac{1}{10}$ +1

$\alpha_6 = 0$

not Support vectors.