Parameter Estimation for HMMs

Example: let both the emitted sequence $\vec{x}$ and the hidden state sequence $\vec{z}$ be known, but the transition and emission parameters be unknown. Let $K = 2$ and $B = 2$, so two hidden states {0,1} and two possible observations {0,1}. Let

$$\vec{z} = [1,\ 0,\ 0,\ 1,\ 0,\ 1,\ 1,\ 0,\ 0,\ 0]$$
$$\vec{x} = [0,\ 1,\ 0,\ 0,\ 1,\ 0,\ 1,\ 1,\ 1,\ 0]$$

1. Warmup: what is L?

2. Let

$$A_{kl} = \#\text{ of transitions from } k \to l \text{ in the data}$$
$$E_k(b) = \#\text{ of emissions of } b \text{ from state } k \text{ in the data}$$

   Fill in the tables below for A_{kl} (row: start state, col: end state) and $E_k(b)$ (row: hidden state, col: emitted state).

A	$l = 0$	$l = 1$
$k = 0$		
$k = 1$		

E	$b = 0$	$b = 1$
$k = 0$		
$k = 1$		

3. To estimate the transition probabilities a_{kl} and emission probabilities $e_k(b)$, we will divide each of the counts above by the sum of the counts in each row:

$$a_{kl} = \frac{A_{kl}}{\sum_{l'} A_{kl'}}\ , \qquad e_k(b) = \frac{E_k(b)}{\sum_{b'} E_k(b')}$$

   Use this idea to fill in the tables for a_{kl} and $e_k(b)$.

a	$l = 0$	$l = 1$
$k = 0$		
$k = 1$		

e	$b = 0$	$b = 1$
$k = 0$		
$k = 1$		

4. What could go wrong with this estimation procedure if we don't observe some transitions and/or emissions?

5. How could you estimate initial probabilities (π_k for each state k) as well?