Viterbi Algorithm for HMMs

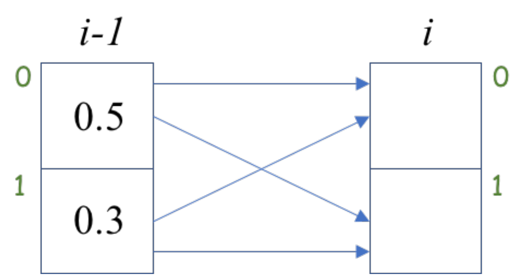
1. Let K be the total number of hidden states and B be the number of possible emissions. Let L be the length of the Markov chain (as well as the length of the emitted sequence). Given the emitted sequence $\vec{x}$, how many hidden state sequences $\vec{z}$ are possible?

2. What is the runtime of the Viterbi algorithm, in terms of K, B, and L?

3. *Example*: Let the table on the left be a representation of the transition probabilities in a Markov chain with $K = 2$ states (0 and 1). Let the rows represent the start state and the columns represent the end state. Fill in the rest of the transition probabilities. Then use them to complete one step of the Viterbi DP table on the right (i.e. label each arrow with its transition probability and then fill in the missing cells). Highlight the arrows we will keep for our traceback.

   Note: state labels are in green below, so $V_0(i-1) = 0.5$ and $V_1(i-1) = 0.3$.

start state \ end state	0	1
0	0.2	
1		0.4



4. Does the sum of each column in our DP table have to sum to 1? Why or why not?

5. Take the log of both sides of our Viterbi recursion. What do you get?