Tree Metrics and Ultrametrics

1. You are given the dissimilarity map δ below for the samples $\mathcal{X} = \{A, B, C, D\}$. Is δ an ultrametric? Why or why not?

δ	A	B	C	D
A	0	3	4	4
B		0	5	5
C			0	2
D				0

2. Is δ a tree metric? If yes, demonstrate the tree metric by constructing a topology and edge weights that match δ. If not, explain why not.

3. Verify that the dissimilarity map d below is an ultrametric. Then confirm that when UPGMA is applied to this matrix, the induced tree metric is equal to the original.

d	P	Q	R	S
P	0	8	12	4
Q		0	12	8
R			0	12
S				0