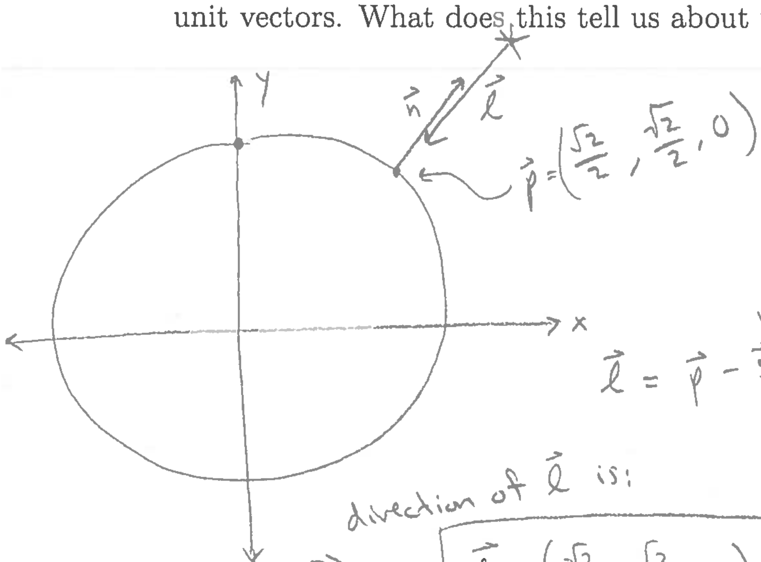
Solutions

Review problems

1. **Lighting:** You are given a unit sphere centered at the origin and light source specified with the following code:

```
var pointLight = new THREE.PointLight("white", 2, 30); // color, intensity, distance light travels
pointLight.position.set( 2, 2, 0 );
```

What is the unit normal vector ($\vec{n}$) at the point $\vec{p} = (\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}, 0)$ on the sphere? What is the unit direction of the light ray ($\vec{l}$) pointing towards this point? Compute the dot product of these two unit vectors. What does this tell us about the color of sphere at this point?



$$\vec{n} = \vec{p} - \vec{c} = \begin{bmatrix} \frac{\sqrt{2}}{2} \\ \frac{\sqrt{2}}{2} \\ 0 \end{bmatrix} - \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} = \underbrace{\begin{bmatrix} \frac{\sqrt{2}}{2} \\ \frac{\sqrt{2}}{2} \\ 0 \end{bmatrix}}_{\text{already unit vector.}}$$

center of sphere

light source

$$\vec{l} = \vec{p} - \vec{s} = \begin{bmatrix} \frac{\sqrt{2}}{2} \\ \frac{\sqrt{2}}{2} \\ 0 \end{bmatrix} - \begin{bmatrix} 2 \\ 2 \\ 0 \end{bmatrix} = \begin{bmatrix} \frac{\sqrt{2}}{2} - 2 \\ \frac{\sqrt{2}}{2} - 2 \\ 0 \end{bmatrix} \approx \begin{bmatrix} -1.3 \\ -1.3 \\ 0 \end{bmatrix}$$

direction of $\vec{l}$ is:

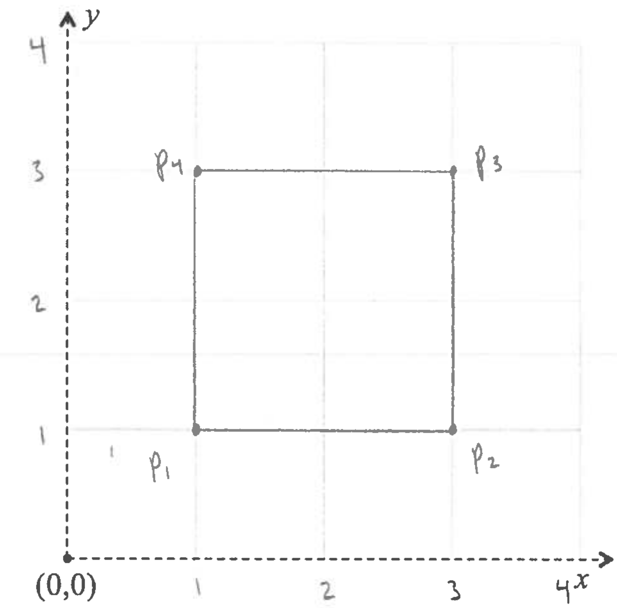
$\Rightarrow \boxed{\vec{l} = \left(-\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}, 0\right)} \Rightarrow \boxed{\vec{n} \cdot \vec{l} = -\frac{\sqrt{2}}{2} \cdot \frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2} \cdot \frac{\sqrt{2}}{2} = -1} \Rightarrow$ pixel very bright!

2. **Projection:** (*adapted from the Fall 2015 final exam*) You are given the following 8 vertices of a cube in world space, a camera at the origin, and a viewport at $z = -1$.

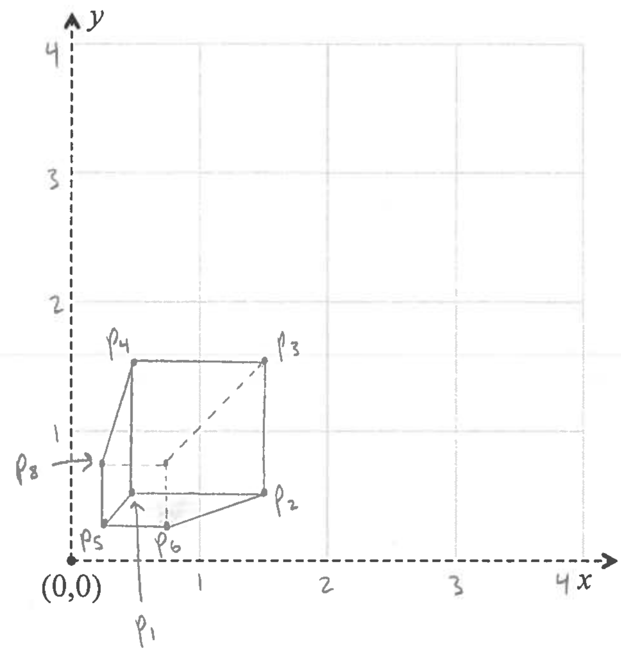
   (a) Fill in the table below with the 2D viewport coordinates for each type of projection. Assume the viewport is large enough that no points will be clipped out.

world coordinates	orthographic projection	perspective projection
p_1 = (1,1,-2)	(1, 1)	(1/2, 1/2)
p_2 = (3,1,-2)	(3,1)	(3/2, 1/2)
p_3 = (3,3,-2)	(3, 3)	(3/2, 3/2)
p_4 = (1,3,-2)	(1, 3)	(1/2, 3/2)
p_5 = (1,1,-4)	(1, 1)	(1/4, 1/4)
p_6 = (3,1,-4)	(3, 1)	(3/4, 1/4)
p_7 = (3,3,-4)	(3, 3)	(3/4, 3/4)
p_8 = (1,3,-4)	(1, 3)	(1/4, 3/4)

(b) Draw what the "viewer" would see in each case (only one quadrant of the viewport is shown).



Orthographic



Perspective

3. Name the algorithm based on the pseudocode below:

```
for all p in pixels:
    create a ray r from camera to p
    for all o in objects of the world:
        calculate intersection of o with r
        keep if closest
    color p based on material of o & angle of surface to light
```

ray - tracing

4. **Ray tracing:** (*adapted from the Fall 2015 final exam*) You are trying to figure out whether a circular mirror on a wall in your scene is visible from a certain pixel. The direction of the ray and the point on the viewport are, respectively:

$$\vec{R}_d = \left(\frac{3}{5}, 0, -\frac{4}{5}\right), \qquad \text{and} \qquad \vec{p}_v = (-1, 2, -1) \qquad (z = -1 \text{ included for clarity}).$$

(a) The camera is 5 units away from the point $\vec{p}_v$. Where is the camera located (find $\vec{R}_0$)?

$$\vec{R}(t) = \vec{R}_0 + t\vec{R}_d \quad \Rightarrow \quad \begin{aligned} c_x &= -1 - 3 = -4 \\ c_y &= 2 - 0 = 2 \\ c_z &= -1 + 4 = 3 \end{aligned}$$

$$\begin{bmatrix} -1 \\ 2 \\ -1 \end{bmatrix} = \begin{bmatrix} c_x \\ c_y \\ c_z \end{bmatrix} + 5 \begin{bmatrix} 3/5 \\ 0 \\ -4/5 \end{bmatrix}$$

$\boxed{\vec{R}_0 = (-4, 2, 3)}$

camera location

(b) The circular mirror is located on the wall represented by the plane $x = 5$. What are the coordinates of $\vec{p}_w$, the point where this ray intersects the wall? How far away is $\vec{p}_w$ from the camera?

$$\vec{R}(t) = \begin{bmatrix} -4 \\ 2 \\ 3 \end{bmatrix} + t \begin{bmatrix} 3/5 \\ 0 \\ -4/5 \end{bmatrix}$$

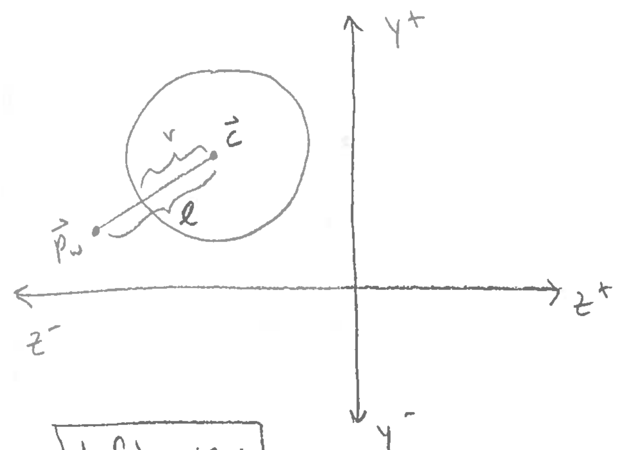
x-coord: $x = 5 = -4 + t \cdot \frac{3}{5} \Rightarrow t \cdot \frac{3}{5} = 9 \Rightarrow \boxed{t = 15}$

y-coord: $y = 2 + t \cdot 0 \Rightarrow \boxed{y = 2}$

z-coord: $z = 3 + 15\left(-\frac{4}{5}\right) = 3 - 12 = \boxed{-9}$

$\Rightarrow \boxed{\vec{p}_w = (5, 2, -9)}$

(c) The circular mirror has center point $\vec{c} = (5, 5, -7)$ and radius $r = 3$. Does this ray intersect the mirror? Justify your answer (it might be helpful to draw a picture of the wall).



left view

Distance between $\vec{p}_w$ and $\vec{c}$ should be less than r if ray intersects mirror:

$$\|\vec{p}_w - \vec{c}\| = \sqrt{(5-5)^2 + (2-5)^2 + (-9 - -7)^2}$$
$$= \sqrt{0 + 9 + 4} = \sqrt{13} = \ell$$

$\sqrt{13} > 3 = r \Rightarrow \boxed{\text{no}}$ outside circle

(d) Create a general algorithm for determining whether a ray intersects a circular object lying on a given plane. You don't need to use code or pseudocode, just a general description of how to find the equivalent of the point on the plane $\vec{p}_w$, and then an inequality in terms of $\vec{p}_w$, $\vec{c}$, and r.

First, find the intersection point $\vec{p}_w$ of the ray and the plane the circle lies on. Then, to test whether the point is inside the circle, check:

$$\boxed{\|\vec{p}_w - \vec{c}\| \le r}$$

→ if $\boxed{\text{no}}$, no intersection with circle

→ if $\boxed{\text{yes}}$, ray does intersect circle.