

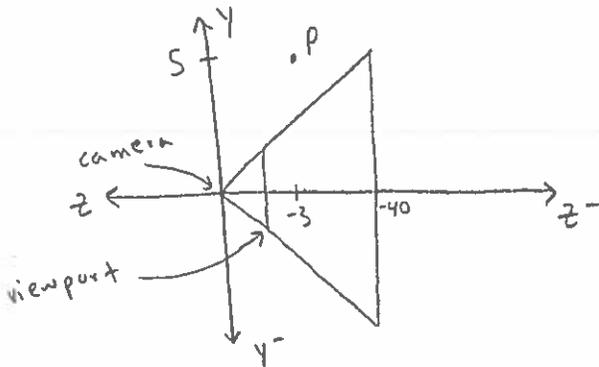
Solutions

Review problems

1. **Perspective projection:** Given a camera defined by the line:

```
var camera = new THREE.PerspectiveCamera( 90, 1, 1, 40 );
```

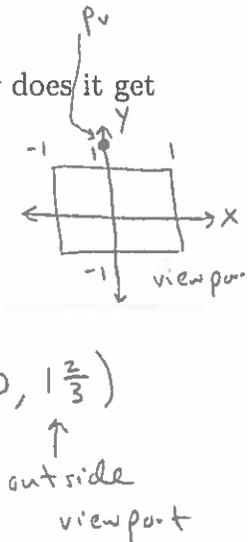
Does the point with world coordinates $p = (0, 5, -3)$ lie inside the viewing frustum or does it get clipped out? *Hint: draw out the right view.*



p is clipped out

another method:

$$p_v = \left(\frac{0}{3}, \frac{5}{3} \right) = \left(0, 1\frac{2}{3} \right)$$



2. **Hidden surface removal (Z-buffering):** You are given two points in the world: a red point at $p_w = (-10, 5, -15)$ and a blue point at $q_w = (-8, 4, -12)$. The camera is defined with the line:

```
var camera = new THREE.PerspectiveCamera( 90, 1, 1, 40 );
```

(a) What are the viewport coordinates (p_v and q_v) of each of these points?

$$p_v = \left(\frac{-10}{15}, \frac{5}{15} \right)$$

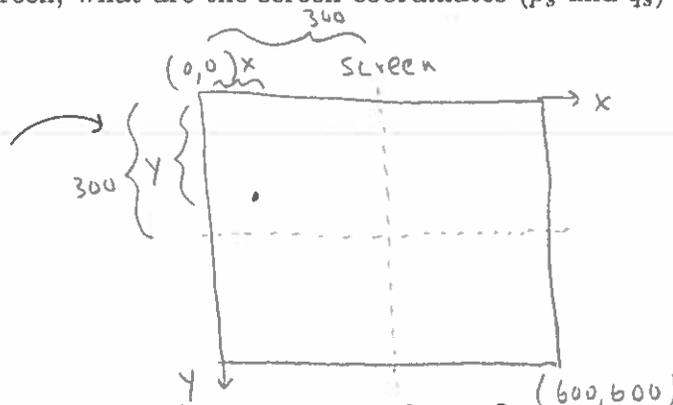
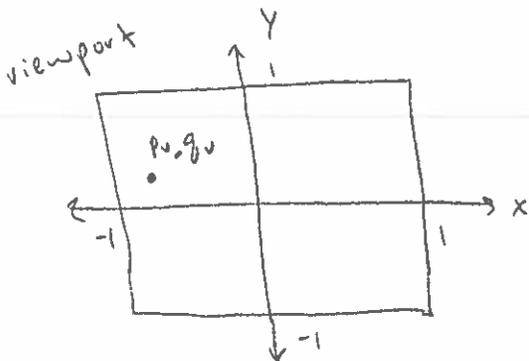
$$q_v = \left(\frac{-8}{12}, \frac{4}{12} \right)$$

$$p_v = \left(-\frac{2}{3}, \frac{1}{3} \right)$$

$$q_v = \left(-\frac{2}{3}, \frac{1}{3} \right)$$

} same viewport coordinates

(b) Given a 600×600 pixel screen, what are the screen coordinates (p_s and q_s) of these points?



$$\frac{y}{300} = \frac{2}{3}$$

$$\Rightarrow y = 200$$

$$\frac{x}{300} = \frac{1}{3}$$

$$x = 100$$

(c) Which color should the viewer see at these screen coordinates?

compare z-coordinates

$$-15 < -12 \Rightarrow \text{pixel at } (100, 200)$$

$$p_w.z < q_w.z$$

should be blue

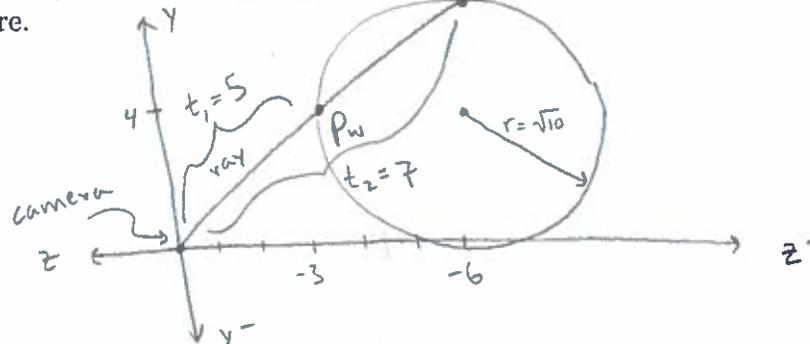
$$\Rightarrow p_s = q_s = (100, 200)$$

3. **Ray-sphere intersection:** In this problem we will determine if a given ray intersects a sphere object in the world. Let the color of the sphere be green.

(a) Given a camera at the origin and a ray $\vec{R}_d = (0, 4/5, -3/5)$, write out our ray equation.

$$R(t) = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} + t \begin{bmatrix} 0 \\ 4/5 \\ -3/5 \end{bmatrix}$$

(b) Let the equation of the sphere be $x^2 + (y - 3)^2 + (z + 6)^2 = 10$, so the center of the sphere is at $\vec{s} = (0, 3, -6)$ and $r = \sqrt{10}$. Draw this setup from the right view, showing the camera, ray, and sphere.



(c) To find out whether this ray intersects this sphere, we need to set up an equation that we can solve for t . For a sphere, this will be a quadratic equation: $at^2 + bt + c = 0$. Compute a , b , and c , where:

i. $a = \|\vec{R}_d\|^2$ (Hint: $\|\vec{R}_d\|$ is a unit vector.)

$$a = 1$$

ii. $b = 2\vec{R}'_0 \cdot \vec{R}_d$, where $\vec{R}'_0 = \vec{R}_0 - \vec{s}$

$$\vec{R}'_0 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} - \begin{bmatrix} 0 \\ 3 \\ -6 \end{bmatrix} = \begin{bmatrix} 0 \\ -3 \\ 6 \end{bmatrix}$$

$$b = 2 \cdot \begin{bmatrix} 0 \\ -3 \\ 6 \end{bmatrix} \cdot \begin{bmatrix} 0 \\ 4/5 \\ -3/5 \end{bmatrix} = 2 \left(0 \cdot 0 + \frac{-12}{5} + \frac{-18}{5} \right) = 2 \left(-\frac{30}{5} \right)$$

iii. $c = \|\vec{R}'_0\|^2 - r^2$

$$c = (0 + 9 + 36) - 10 \Rightarrow c = 35$$

$$\Rightarrow b = -12$$

Put these together to obtain a quadratic equation in terms of t .

$$t^2 - 12t + 35 = 0$$

(d) Solve for t , using either the quadratic formula or factoring. Interpret the t value(s).

$$\left. \begin{array}{l} 35 = 5 \cdot 7 \\ -12 = -5 + -7 \end{array} \right\} t^2 - 12t + 35 = (t - 5)(t - 7) = 0 \left\{ \begin{array}{l} t_1 = 5 \\ t_2 = 7 \end{array} \right.$$

(e) For this ray, find the world coordinates of the point that should be shown on the screen.

$$R(5) = 5 \cdot \begin{bmatrix} 0 \\ 4/5 \\ -3/5 \end{bmatrix} \Rightarrow P_w = (0, 4, -3)$$

(f) Say another object is in this world, and after finding its intersection point with this ray, a value of $t = -3$ is obtained. Let the color of this object be pink. For this ray, should a pink pixel be shown instead of green one, since this object has a smaller t value?

negative t means behind the camera

no!