Solutions

Ray-tracing

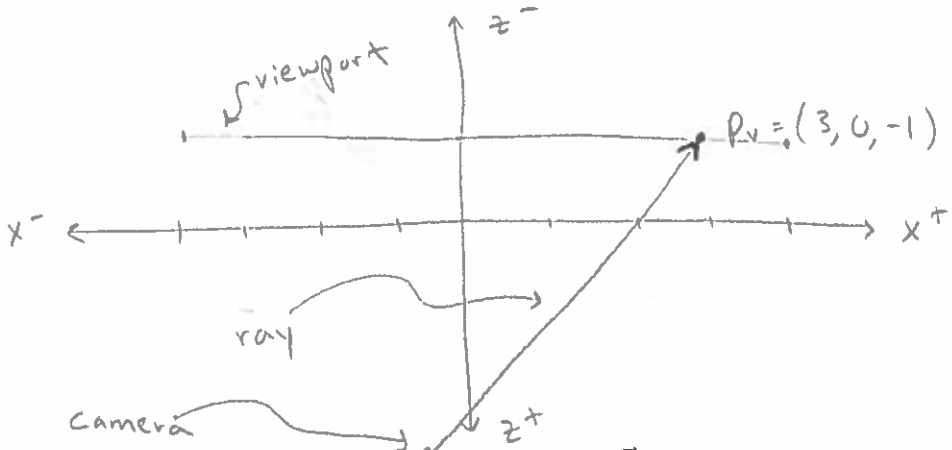
1. You are given a ray with direction $\vec{R}_d = (\frac{\sqrt{2}}{2}, 0, -\frac{\sqrt{2}}{2})$, pointing towards a point with viewport coordinates $p_v = (3, 0, -1)$. Assume the viewport is 8x8 units, centered at the origin.

   (a) Verify that $\vec{R}_d$ is a unit vector.

   $$\|\vec{R}_d\| = \sqrt{\left(\frac{\sqrt{2}}{2}\right)^2 + \left(-\frac{\sqrt{2}}{2}\right)^2} = \sqrt{\frac{2}{4} + \frac{2}{4}} = \sqrt{\frac{4}{4}} = \boxed{1} \quad \checkmark$$

   (b) What view (right, left, top, bottom, back, front) would be most helpful for drawing this setup? Draw this view below, showing the viewport, ray, and point.

   TOP



note: in our normal setup, the camera would be somewhere along the z-axis

   (c) What is the location of the camera $\vec{R}_0 = (c_x, c_y, c_z)$ if it is $t = 6$ units away from this point?

$$\begin{bmatrix} 3 \\ 0 \\ -1 \end{bmatrix} = \begin{bmatrix} c_x \\ c_y \\ c_z \end{bmatrix} + 6 \begin{bmatrix} \frac{\sqrt{2}}{2} \\ 0 \\ -\frac{\sqrt{2}}{2} \end{bmatrix} \Rightarrow$$

$3 = c_x + 3\sqrt{2} \Rightarrow c_x = 3 - 3\sqrt{2}$

$\boxed{c_x \approx -1.24}$

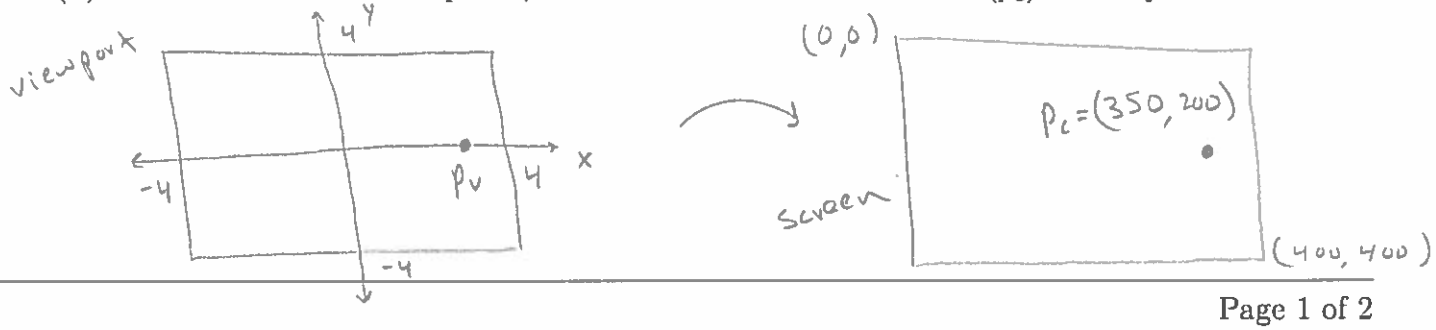
$0 = c_y + 6 \cdot 0 \Rightarrow \boxed{c_y = 0}$

$-1 = c_z - 3\sqrt{2} \Rightarrow c_z = -1 + 3\sqrt{2}$

$\boxed{c_z \approx 3.24}$

$\Rightarrow$ camera at $\boxed{(-1.24, 0, 3.24)}$

   (d) If the screen is 400x400 pixels, what are the screen coordinates (p_c) of this point?



2. **Determining the direction of a ray:** Given a camera positioned at $\vec{R}_0 = (6, -3, 2)$ and a point on the viewport at $p_v = (2, -3, -1)$, determine the unit vector in the direction of the ray, $\vec{R}_d$.

$$p_v - \vec{R}_0 = \begin{bmatrix} 2 \\ -3 \\ -1 \end{bmatrix} - \begin{bmatrix} 6 \\ -3 \\ 2 \end{bmatrix} = \begin{bmatrix} -4 \\ 0 \\ -3 \end{bmatrix} \qquad \text{length} = 5$$

$$\Rightarrow \quad \vec{R}_d = \begin{bmatrix} -4/5 \\ 0 \\ -3/5 \end{bmatrix}$$

3. **Ray-plane intersection:** Say there is a "side wall" in the world, represented by a plane with equation $x = -2$. Where does the ray from (2) intersect this plane? Call this point p_w. How far is p_w from the camera (i.e. what is t)? *Hint: find t first, then p_w.*

$$p_w = \begin{bmatrix} -2 \\ y \\ z \end{bmatrix} = \begin{bmatrix} 6 \\ -3 \\ 2 \end{bmatrix} + t \cdot \begin{bmatrix} -4/5 \\ 0 \\ -3/5 \end{bmatrix} \Rightarrow \quad -2 = 6 + t\left(-\tfrac{4}{5}\right)$$

$$-8 = t\left(-\tfrac{4}{5}\right)$$

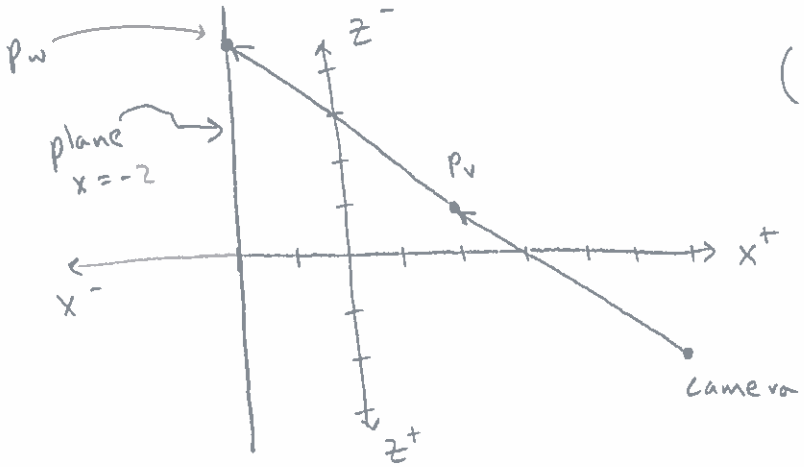
$$\Rightarrow \boxed{t = 10}$$

$$\Rightarrow \quad y = -3 + 10 \cdot 0 \Rightarrow \boxed{y = -3}$$

$$z = 2 + 10\left(-\tfrac{3}{5}\right) \Rightarrow z = 2 - 6 \Rightarrow \boxed{z = -4}$$

$$\boxed{p_w = (-2, -3, -4)}$$

4. **Visualization:** Draw a picture of this setup from the "top" view. Label all the points. Does this visually agree with your answers from (2) and (3)? (yes)



(y-coordinates ignored)