Ray-plane intersection

When using a ray-tracing approach to rendering, it is important to determine whether or not each ray intersects with a given object. In this example we will intersect a ray with a plane.

(a) Setup: here we will have a "ground" plane spanning the entire world at $y = -6$. We will have a camera at the origin and a screen that is 400×400 pixels. Our aspect ratio will be 1, fov=90°, and `near` $= 1$. This will give us a viewport at $z = -1$, with $x_{\min} = -1$, $x_{\max} = 1$, $y_{\min} = -1$, $y_{\max} = 1$, so the origin of the viewport is at the center of the screen. We will cast a ray from the camera through the pixel (200, 350).

First, draw the *right view* of this setup, labeling your axes, the viewport, the camera, the ray, the plane, and the intersection point of the ray with the plane.

(b) In the next part we will build up our parametric ray equation:

$$\vec{R}(t) = \vec{R}_0 + t\vec{R}_d$$

where $\vec{R}_0$ is the starting point of the ray, $\vec{R}_d$ is the unit vector pointing in the direction of the ray, and $t \in [0, \infty)$. Write down $\vec{R}_0$ and $\vec{R}_d$, showing all your work for $\vec{R}_d$. Put this together to form the ray equation for this specific ray.

(c) Next, find the intersection point $p = (x, y, z)$ of this ray with the plane. Show your steps clearly, including finding the value of t at the intersection point.

(d) From your result above, what constraints should be placed on z_{far} so that some of the plane is visible on the screen?